DOCUMENT RESUME

ED 238 681 SE 041 781

AUTHOR Johnston, Patsy
TITLE Mathematics Enrichment: Grade 4. Curriculum Bulletin Number 236.
INSTITUTION Fort Worth Independent School District, TX. Div. of Curriculum Development.
PUB DATE 81
NOTE 104p.; For related document, see SE 041 782.
AVAILABLE FROM Fort Worth Independent School District, 3210 West Lancaster, Fort Worth, TX 76107 ($15.00).
PUB TYPE Guides - Classroom Use - Materials (For Learner) (051)

EDRS PRICE MF01 Plus Postage. PC Not Available from EDRS.
DESCRIPTORS *Elementary School Mathematics; *Enrichment Activities; Grade 4; Intermediate Grades; Learning Activities; *Mathematical Concepts; *Mathematical Enrichment; Mathematics Instruction; Mathematics Materials; Puzzles; Reinforcement
IDENTIFIERS PF Project

ABSTRACT

Enrichment activities for fourth-grade mathematics are presented. Some of the activities reinforce principles taught in the regular program; others introduce new concepts to challenge students. The activities are divided into the following categories: number pictures; multiplying or dividing by 10, 100, or 1000; tic-tac-toe word problems; map coloring; number tricks; logic puzzles; crossnumber puzzles; letters make words and words make cents (making words out of letters with differing point values); suppressed digits; Victorian arithmetic; graphing; geometric shapes; spatial perception; word problems (advertising puzzles); finding the square root of a number; and number sense (patterns in multiplication and division). Answers to the puzzles and other activities are appended. (DC)

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ED238681

MATHEMATICS ENRICHMENT

Grade 4

Curriculum Bulletin Number 236
Fort Worth Independent School District
Fort Worth, Texas

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Foreword

These activities were written for the enrichment of the fourth-grade mathematics program. They are to be used throughout the year to create interest and enjoyment in mathematics. Some of the materials are for reinforcement of principles taught in the regular program; others introduce new concepts to challenge the pupils.

The project development was under the direction of Crawford Johnson, Program Director for Mathematics, and J. D. Shipp, Director of Elementary Schools. Patsy Johnston wrote and edited these activities. The contributions of everyone concerned with the development of these activities are sincerely appreciated.

This publication was planned, written, edited, and published in the Department of Curriculum Development.

Dewey W. Mays, Jr.

Dewey W. Mays, Jr.
Director of Curriculum Development

August, 1981

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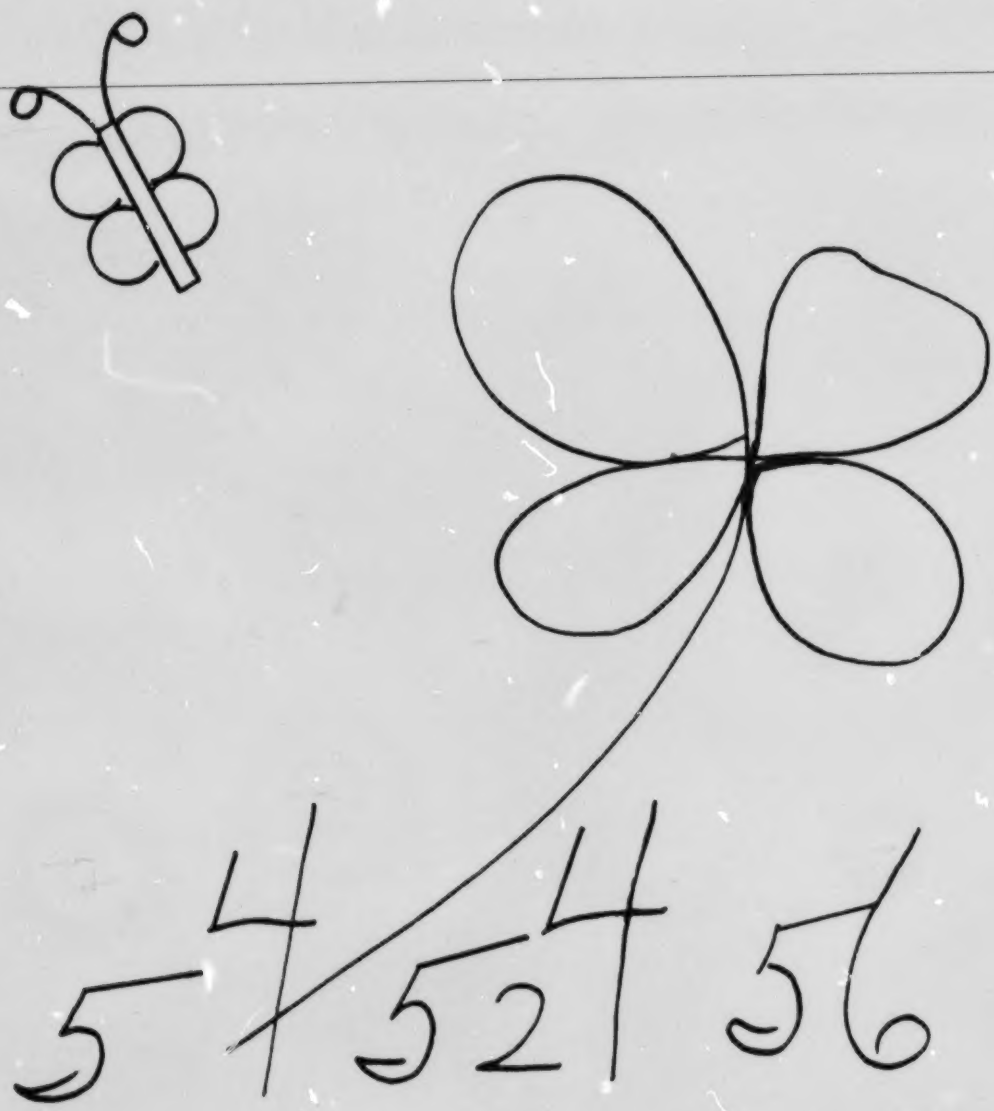
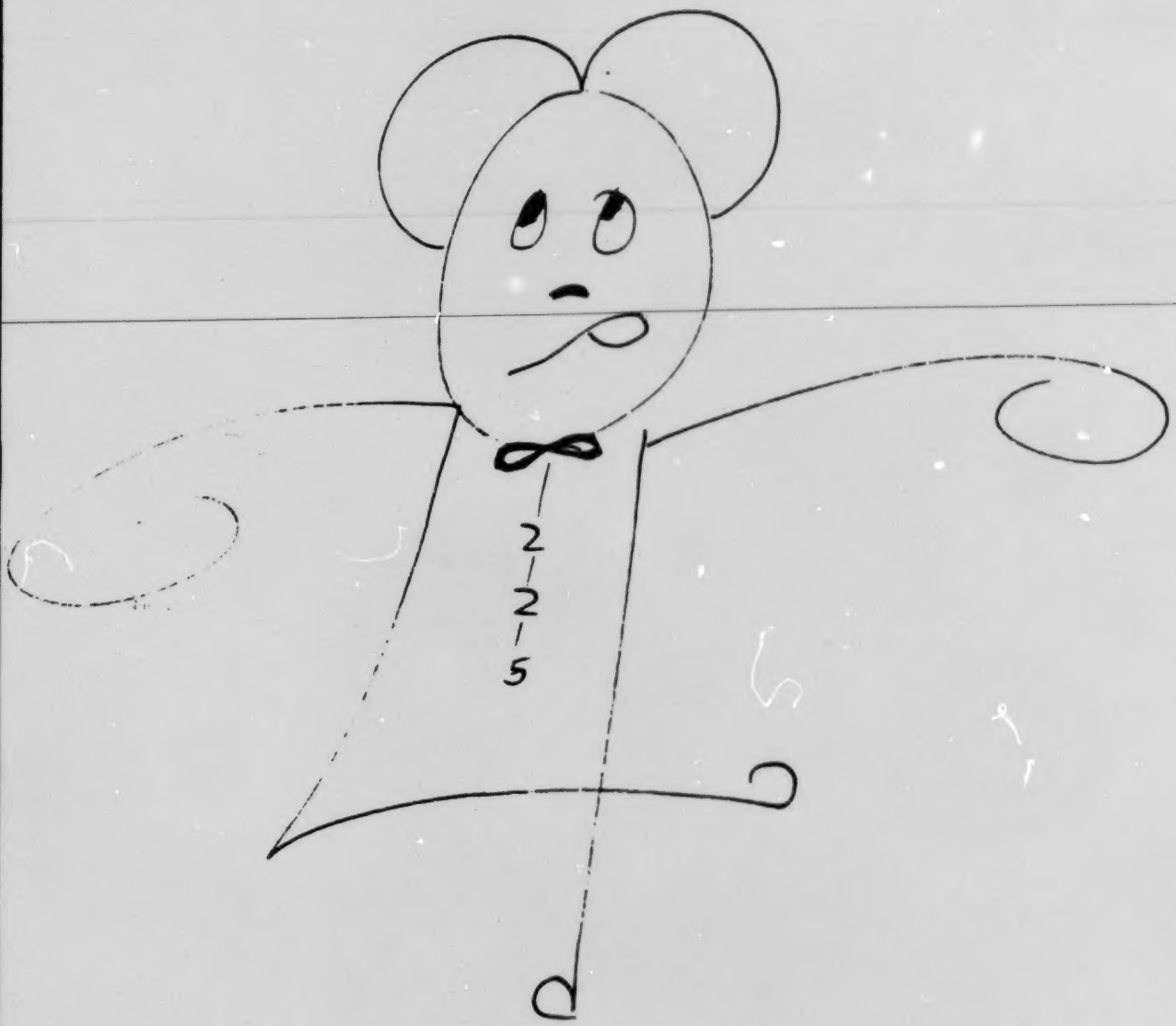
NUMBER PICTURES

PICTURES CAN BE MADE FROM NUMBERS. SEE IF YOU CAN MAKE ONE.



9
5
2
3
1
8
7
0

8


2
2
5

MULTIPLYING OR DIVIDING BY 10, 100, OR 1000

RULE: To multiply a number:

A) by 10, move its decimal point one place to the right;

B) by 100, move its decimal point two places to the right;

C) by 1000, move its decimal point three places to the right.

REMEMBER: When multiplying by 10, 100, or 1000, the answer must be larger than the original number. The decimal point will be moved the same number of places as there are zeros in the multiplier.

EXAMPLE: Multiply 25 by 10. Ten has one zero; therefore, the decimal will be moved one place. If a number has no decimal point printed, the decimal point is understood to to be after the last number. 25--25.--250. Check to see if 250 is larger than 25.

RULE: To divide a number:

A) by 10, move its decimal point one place to the left;

B) by 100, move its decimal point two places to the left;

C) by 1000, move its decimal point three places to the left.

REMEMBER: When dividing by 10, 100, or 1000, the answer must be smaller than the original number. The decimal point will be moved the same number of places as there are zeros in the divisor.

EXAMPLE: Divide 327 by 100. One hundred has two zeros; therefore, the decimal will be moved two places. If the number has no decimal point printed, the decimal point is understood to be after the last number. 327--327.--3.27. Check to see if 3.27 is smaller than 327.

PROBLEMS

1. Mr. Gardner harvested 207.5 bu. of soybeans from a ten-acre field. What was the yield per acre?

2. Mr. Carter harvested 6435 lb. of peanuts from a ten-acre field.
   a) What was the average yeld from an acre?
   b) What would the yield be for 100 acres at that rate?

3. If it cost $9.75 to have 1000 tickets printed for a football game, how much would 100 cost at the same rate?

4. Sugar is selling for $51.90 for 100 lb.
   a) What does that average per pound?
   b) What would 1000 lb. cost?

5. In a United Way drive, $2532.15 was collected from 100 teachers in a school. What did this average per person?

6. Birchwood weighs 3670 per 1000 board feet.
   a) How much will 1 board foot weigh?
   b) How much will 100 board feet weigh?

7. At Great Salt Lake, water is run into reserviors and evaporated, leaving salt, which weighs approximately 134 lb. a cubic foot.
   a) How much will 100 cubic-foot blocks weigh?
   b) How much will 1000 cubic-foot blocks weigh?
   c) How much will 10,000 cubic-foot blocks weigh?

8. Twelve boys agreed to rent a motorboat, each paying $1.50. When the day for the outing arrived, only ten of the boys could go. They agreed to divide the expense equally; how much did each pay?

TIC-TAC-TOE

CHOOSE ANY PROBLEM BELOW. SOLVE IT. PUT AN X OR A O ON THE ANSWER ON THE TIC-TAC TOE BOARD. IF TWO PEOPLE PLAY, THREE Xs OR Os IN A LINE WIN. IF ONE PERSON PLAYS, TWO TIC-TAC-TOES ARE NEEDED TO WIN.

2.16	12.25	20
1575	.87	490
70	.50	.40

1. MELBA BOUGHT 2 PACKAGES OF RAISINS FOR $.20 EACH. PHILIP BOUGHT 3 PACKAGES OF SUNFLOWER SEEDS FOR $.29 EACH.
   A) HOW MUCH DID MELBA SPEND FOR THE RAISINS?
   B) HOW MUCH DID PHILIP SPEND FOR THE SUNFLOWER SEEDS?

2. MAVIS EARNS $4 A DAY ON WEEKDAYS AND $6 A DAY ON SATURDAYS. LAST MONTH SHE WORKED 13 WEEKDAYS AND 3 SATURDAYS. HOW MUCH DID SHE EARN?

3. WILLIAM SOLD 12 PENCILS FOR 8¢ A PIECE. HE ALSO SOLD 8 ERASERS FOR 15¢ A PIECE. WHAT WERE HIS TOTAL SALES FOR THE PENCILS AND ERASERS?

4. LEROY HAS 82 CORDS OF FIREWOOD TO SELL FOR $70 A CORD. IF HE SELLS 75 CORDS, WHAT IS THE VALUE OF THE FIREWOOD THAT IS LEFT?

5. IF 3 KG OF HAMBURGER COST $7.35, HOW MUCH WOULD 5 KG COST?

6. CECILIA SAVED $10.25 THE FIRST WEEK, AND $9.75 THE SECOND WEEK.
   A) HOW MUCH HAS SHE SAVED IN ALL?
   B) HOW MUCH MORE MONEY DID SHE SAVE THE FIRST WEEK THAN THE SECOND?

7. FENCING COSTS $3.50 A METER. HOW MUCH WOULD IT COST TO FENCE IN A FIELD THAT IS 140 M LONG AND 85 M WIDE?

TIC-TAC-TOE

CHOOSE ANY PROBLEM BELOW. SOLVE IT. PUT AN X OR A O ON THE ANSWER ON THE TIC-TAC-TOE BOARD. IF TWO PEOPLE PLAY, THREE Xs OR Os IN A LINE WIN. IF ONE PERSON PLAYS, TWO TIC-TAC-TOES ARE NEEDED TO WIN.

1888	675	1788
450	1850	12
648	30	817

1. PHILIP BOUGHT A HAMBURGER FOR $.89, FRENCH FRIES FOR $.49, AND A SODA FOR $.45. WHAT IS HIS CHANGE FROM A $10 BILL?

2. MAXINE HAS FILLED 59 PAGES OF HER STAMP BOOK. EACH PAGE HAS 32 STAMPS. HOW MANY STAMPS DOES SHE HAVE IN ALL?

3. AT THE SOCCER STADIUM, THERE ARE 5766 SEATS. ONLY 3978 SEATS ARE TAKEN. HOW MANY SEATS ARE LEFT?

4. THERE ARE 27 CLASSROOMS IN THE BUILDING. EACH ROOM HAS ABOUT 24 STUDENTS. HOW MANY STUDENTS ARE THERE IN THE BUILDING?

5. FELECIA BOUGHT 6 POSTERS. EACH POSTER COSTS $2. SHE ALSO BOUGHT TWO PICTURE FRAMES. EACH FRAME COSTS $15.
   A) HOW MUCH DID SHE PAY FOR THE POSTERS?
   B) HOW MUCH DID SHE PAY FOR THE FRAMES?

6. AN OPERA COMPANY BUDGETED $2000 FOR A PRODUCTION. THEY BOUGHT 9 COSTUMES FOR $75 EACH, $300 FOR SCENERY, AND $150 FOR MUSIC.
   A) HOW MUCH DID THEY PAY FOR THE COSTUMES?
   B) HOW MUCH DID THEY PAY FOR THE SCENERY AND MUSIC?
   C) HOW MUCH MONEY DO THEY HAVE LEFT AFTER PAYING ONLY FOR THE MUSIC.

TIC-TAC-TOE

CHOOSE ANY PROBLEM BELOW. SOLVE IT. PUT AN X OR A O ON THE ANSWER ON THE TIC-TAC-TOE BOARD. IF TWO PEOPLE PLAY, THREE Xs OR Os IN A LINE WIN. IF ONE PERSON PLAYS, TWO TIC-TAC-TOES ARE NEEDED TO WIN.

2	8	5
7	6	3
4	9	10

1. ADD 7 TO MY NUMBER AND YOU GET 15. WHAT'S MY NUMBER?

2. SUBTRACT 6 FROM MY NUMBER AND YOU GET 3. WHAT'S MY NUMBER?

3. SUBTRACT 5 FROM MY NUMBER. THEN ADD 8 AND YOU GET 10. WHAT'S MY NUMBER?

4. ADD MY NUMBER TO ITSELF. THEN SUBTRACT 4 AND YOU GET 0. WHAT'S MY NUMBER?

5. SUBTRACT 7 FROM MY NUMBER AND YOU GET 3. WHAT'S MY NUMBER?

6. WHAT NUMBER ADDED TO 7 EQUALS 10?

7. FOUR MORE THAN A CERTAIN NUMBER IS EQUAL TO 8. WHAT IS THAT NUMBER?

8. IF YOU ADD 22 TO A CERTAIN NUMBER, YOU OBTAIN 27. WHAT NUMBER IS THIS?

9. ADD MY NUMBER TO ITSELF AND YOU GET 12. WHAT'S MY NUMBER?

TIC-TAC-TOE

CHOOSE ANY PROBLEM BELOW. SOLVE IT. PUT AN X OR A O ON THE ANSWER ON THE TIC-TAC-TOE BOARD. IF TWO PEOPLE PLAY, THREE Xs OR Os IN A LINE WIN. IF ONE PERSON PLAYS, TWO TIC-TAC-TOES ARE NEEDED TO WIN.

29	9	32
8	7	3
10	30	31

1. ERIK HAD $5 AND SPENT $2 FOR SOME FOOD. WHAT WAS THE TOTAL CHANGE HE RECEIVED?

2. BILL EARNED $25 IN JUNE AND $15 IN JULY. HOW MUCH MORE DID HE EARN IN JUNE THAN IN JULY?

3. THE DIFFERENCE BETWEEN BILL'S MARBLES AND SAM'S IS 4. IF SAM HAS 27 AND HE HAS LESS THAN BILL, HOW MANY MARBLES DOES BILL HAVE?

4. THERE ARE 72 CHAIRS IN A ROOM. THERE ARE 8 CHAIRS IN EACH ROW. HOW MANY ROWS ARE THERE?

5. WHAT NUMBER SUBTRACTED ROM 27 EQUALS 19?

6. ANN CAUGHT 56 INSECTS. HOW MANY JARS DOES SHE NEED IF SHE KEEPS 8 INSECTS IN EACH JAR?

7. HOW MANY HOURS ARE 1740 MINUTES?

8. HOW MANY DAYS IN APRIL?

9. FIND THE PERIMETER OF A SQUARE WITH A SIDE 8 FT.

TIC-TAC-TOE

CHOOSE ANY PROBLEM BELOW. SOLVE IT. PUT AN X OR A O ON THE ANSWER ON THE TIC-TAC-TOE BOARD. IF TWO PEOPLE PLAY, THREE Xs OR Os IN A LINE WIN. IF ONE PERSON PLAYS, TWO TIC-TAC-TOES ARE NEEDED TO WIN.

303	3.94	581
31	500	180
5.04	6	356

1. MR. HEITZ BOUGHT A SHIRT FOR $8.98, AND A PAIR OF PANTS FOR $25.98. HE GAVE THE CLERK $40. HOW MUCH CHANGE DID MR. HEITZ RECEIVE?

2. A PARKING GARAGE HAS 7 FLOORS. EACH FLOOR CAN HOLD 83 CARS. HOW MANY CARS CAN THE GARAGE HOLD IN ALL?

3. CHRISTINE IS PAID $4.50 AN HOUR. LAST WEEK, SHE WORKED 40 HOURS. HOW MUCH DID SHE EARN?

4. 4242 FISH ARE GROUPED INTO EQUAL SCHOOLS OF 14. HOW MANY SCHOOLS OF FISH ARE THERE?

5. IT TAKES VICKI ABOUT 10 MINUTES TO WALK 1 KM. HOW MANY KILOMETERS CAN SHE WALK IN ONE HOUR?

6. MS. RAMIREZ AVERAGES 60 WORDS PER MINUTE WHEN TYPING. ABOUT HOW MANY MINUTES WOULD IT TAKE HER TO TYPE A 30,000 WORD REPORT?

7. JOCELENE BOUGHT SOME RIBBON FOR $3.25, AND A PACKAGE OF HAIRPINS FOR $.69. SHE ALSO BOUGHT 4 PACKAGES OF BARRETTES FOR $.89 EACH.

   A) WHAT WAS THE TOTAL COST FOR THE BARRETTES?
   B) HOW MUCH MORE DID SHE SPEND FOR BARRETTES THAN THE RIBBONS?
   C) HOW MUCH DID SHE SPEND FOR THE RIBBON AND HAIRPINS?

TIC-TAC-TOE

272	1.50	990
1.78	2.20	292
7084	4	16.11

CHOOSE ANY PROBLEM BELOW. SOLVE IT. PUT AN X OR AN O ON THE ANSWER ON THE TIC-TAC-TOE BOARD. IF TWO PEOPLE PLAY, THREE XS OR OS IN A LINE WIN. IF ONE PERSON PLAYS, TWO TIC-TAC-TOES ARE NEEDED TO WIN.

1. MELVIN BOUGHT A RADIO FOR $17.89 AND TWO BATTERIES FOR $.89 EACH.
   A) HOW MUCH DID HE PAY FOR THE BATTERIES?
   B) HOW MUCH MORE DID HE PAY FOR THE RADIO THAN FOR THE BATTERIES?
2. THE DISTANCE FROM ORANGE CITY TO APPLE TOWN IS 4037 KM. THE DISTANCE FROM APPLE TOWN TO PECAN CITY IS 3047 KM.
   A) WHAT IS THE TOTAL DISTANCE FROM ORANGE CITY TO APPLE TOWN AND FROM APPLE TOWN TO PECAN CITY?
   B) HOW MUCH FARTHER IS ONE DISTANCE THAN THE OTHER ONE?
3. FOUR PEOPLE PLAN TO RENT A BOAT FOR $30. THEY WILL SHARE THE COST EQUALLY. HOW MUCH WOULD EACH PERSON SAVE IF THEY CAN FIND A FIFTH PERSON TO SHARE THE COST?
4. LOUISE BOWLED 3 GAMES. EACH GAME WAS 60¢. SHE ALSO RENTED SHOES. THE SHOES COST 40¢. HOW MUCH DID SHE SPEND IN ALL?
5. THERE ARE 24 PEOPLE AT A PICNIC. IF THEY SHARE 48 BOTTLES OF ORANGE SODA, AND 48 BOTTLES OF CHERRY SODA EQUALLY, HOW MANY BOTTLES OF SODA WILL EACH PERSON GET?
6. CHUCK HAS $350 TO SPEND. HE BOUGHT 2 MOTORCYCLE HELMETS. EACH HELMET COSTS $136. HE ALSO BOUGHT A JACKET FOR $58.
   A) AFTER PAYING ONLY FOR THE JACKET, HOW MUCH DOES HE HAVE LEFT?
   B) WHAT WAS THE TOTAL COST OF THE HELMETS?

17

TIC-TAC-TOE

CHOOSE ANY PROBLEM BELOW. SOLVE IT. PUT AN X OR AN O ON THE ANSWER ON THE TIC-TAC-TOE BOARD. IF TWO PEOPLE PLAY, THREE Xs OR Os IN A LINE WIN. IF ONE PERSON PLAYS, TWO TIC-TAC-TOES ARE NEEDED TO WIN.

7	101	1.58
1.52	19	4543
455	13.50	1433

1. LEO'S PLANT IS 76 CM TALL. FLYREE'S PLANT IS 88 CM TALL. GINGER'S PLANT IS 69 CM TALL.
   A) FLYREE'S PLANT IS HOW MUCH TALLER THAN GINGER'S?
   B) LEO'S PLANT IS HOW MUCH TALLER THAN GINGER'S?

2. THE DISTANCE FROM SIDE TOWN TO SANTO IS 2044 KM. THE DISTANCE FROM SANTO TO WELL TOWN IS 2499 KM.
   A) HOW MUCH FARTHER IS THE SECOND DISTANCE THAN THE FIRST?
   B) WHAT IS THE TOTAL DISTANCE?

3. A 0.5 KG BOX OF COOKIES COSTS 79¢. A 1.5 KG BOX OF THE SAME COOKIES COSTS $2.28.
   A) WHAT IS THE COST PER KG OF THE SMALL BOX?
   B) WHAT IS THE COST PER KG OF THE LARGE BOX?

4. JOHNNY PARKED HER CAR IN A PARKING GARAGE FOR 2 DAYS. THE GARAGE COSTS $7 A DAY. AT THE SAME GARAGE, IT COSTS 25¢ A DAY FOR A BICYCLE. HOW MUCH WOULD SHE HAVE SAVED IN PARKING IF SHE HAD RIDDEN HER BICYCLE INSTEAD OF DRIVING HER CAR?

5. AT THE FOOTBALL GAME, THERE WERE 666 FANS FOR THE "HOME" TEAM AND 767 FANS FOR THE "AWAY" TEAM.
   A) HOW MANY MORE FANS DOES ONE TEAM HAVE THAN THER OTHER TEAM?
   B) WHAT WAS THE TOTAL NUMBER OF FANS?

TIC-TAC-TOE

CHOOSE ANY PROBLEM BELOW. SOLVE IT, PUT AN X OR AN O ON THE ANSWER ON THE TIC-TAC-TOE BOARD. IF TWO PEOPLE PLAY, THREE Xs OR Os IN A LINE WIN. IF ONE PERSON PLAYS, TWO TIC-TAC-TOES ARE NEEDED TO WIN.

60	4.28	1789
768	42	.96
6.27	4	2

1. ON A WALKATHON, JAMES WALKED 29 KM AND MARY WALKED 31 KM.
   A) HOW FAR DID THEY WALK IN ALL?
   B) HOW MUCH FARTHER DID MARY WALK?

2. THE CARTERS TRAVELED 768 KM ON MONDAY, 468 KM ON TUESDAY, AND 553 KM ON WEDNESDAY.
   A) HOW MANY KILOMETERS DID THEY TRAVEL IN ALL?
   B) ON THE DAY THEY TRAVELED FARTHERES', HOW FAR DID THEY TRAVEL?

3. RAYBENA HAS 19 BOOKS IN HER LIBRARY. PURAL HAS 23 BOOKS IN HER LIBRARY
   A) HOW MANY MORE BOOKS DOES PURAL HAVE THAN RAYBENA?
   B) HOW MANY BOOKS DO THEY HAVE ALTOGETHER?

4. WADE BOUGHT A RECORD FOR $.99, A CAP FOR $3.29, AND A PAIR OF SOCKS FOR $1.99.
   A) HOW MUCH DID HE SPEND FOR THE RECORD AND CAP?
   B) HOW MUCH DID HE SPEND FOR ALL THREE ITEMS?

5. SYBAL'S ALLOWANCE IS $3.60. SHE SPENDS $1.20 OF HER ALLOWANCE ON A RECORD, AND $1.44 OF HER ALLOWANCE ON A BOOK. HOW MUCH OF HER ALLOWANCE DOES SHE HAVE LEFT?

MAP COLORING

WHEN A MAP IS COLORED, ANY TWO STATES OR REGIONS WHICH ARE NEXT TO ONE ANOTHER MUST BE COLORED DIFFERENTLY.

THE SEA MUST ALSO BE COLORED DIFFERENTLY FROM THE REGION NEXT TO IT.

WHEN THE REGIONS MEET AT A POINT, THEY CAN BE IN THE SAME COLOR.

MAKE A TRACING OF THE MAP IF ONE IS NOT FURNISHED.

COLOR EACH STATE AND THE SEA USING AS FEW COLORS AS POSSIBLE.

WHAT IS THE SMALLEST NUMBER OF COLORS YOU CAN USE?

TRY SEVERAL MAPS USING THE SAME PROCEDURE. WHAT IS YOUR CONCLUSION?

AFRICA

Mediterranean Sea
Morocco
Tunisia
Algeria
Libya
Egypt
Red Sea
Spanish Saharah
Mauritania
Republic of Mali
Niger
Chad
Sudan
Gambia
Guinea-Bissau
Guinea
Upper Volta
Ivory Coast
Ghana
Nigeria
Cameroun
Central African Republic
Ethiopia
Afars Issas
Somali Republic
Sierra Leone
Liberia
Togo
Dahoney
Equitorial Guinea
Gabon
Congo Republic
Zaire
Uganda
Kenya
Lake Victoria
Rwanda
Tanzania
Indian Ocean
Atlantic Ocean
Angola
Zambia
Malawi
Namibia
Rhodesia
Mozambique
Botswana
Malagasy Republic
South Africa

United States
Atlantic Ocean
Gulf of
Mexico
Mexico
Cuba
Dominican
Republic
Haiti
Puerto
Rico
Jamaica
Belize
Honduras
Guatemala
Nicaragua
El
Salvador
Panama Canal
Costa Rica
Panama
Pacific Ocean
South America

18

23

24

19

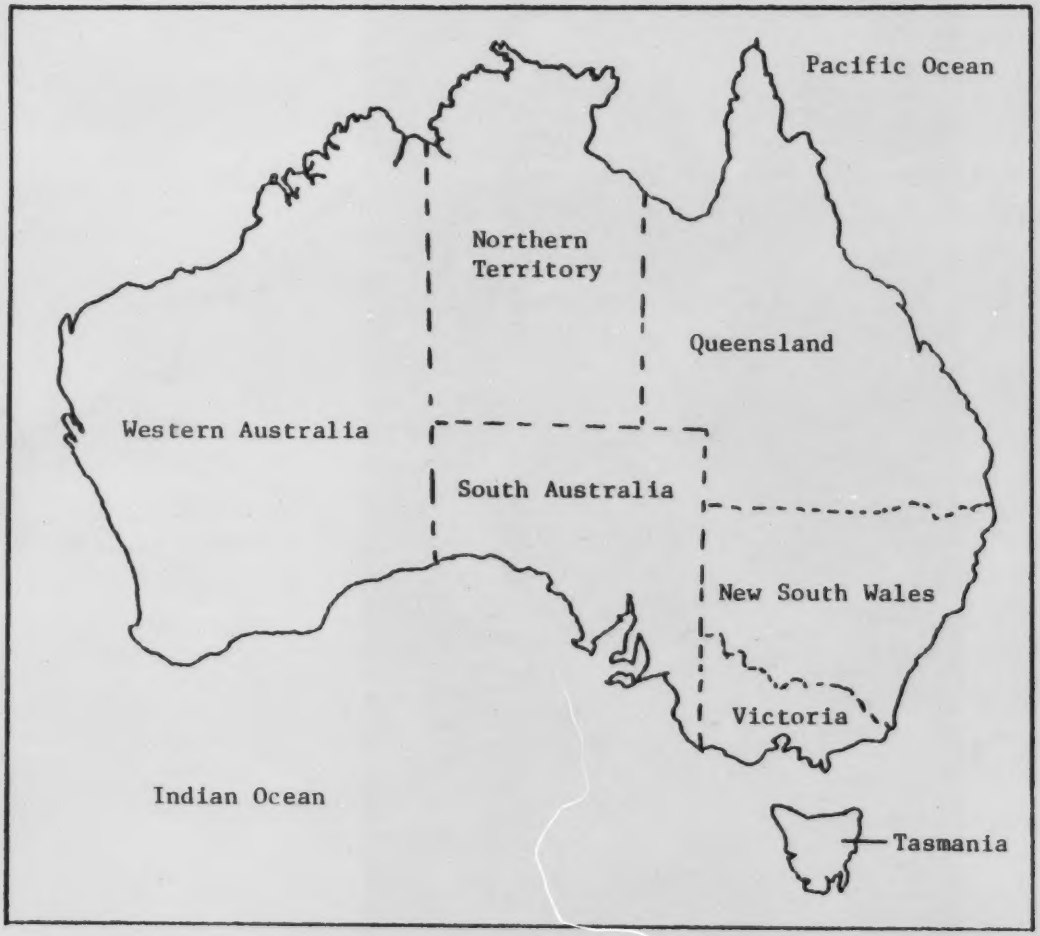
25

26

SOUTH AMERICA

27

AUSTRALIA


Pacific Ocean
Northern Territory
Queensland
Western Australia
South Australia
New South Wales
Victoria
Indian Ocean
Tasmania

CANADA
WASHINGTON
MONTANA
NORTH DAKOTA
OREGON
IDAHO
SOUTH DAKOTA
WYOMING
MINN.
WIS.
MICHIGAN
NEW YORK
VERMONT
MAIN
NEW HAMPSHIRE
MASS
R.I.
CONN.
NEBRASKA
IOWA
PA.
NEW JERSEY
DELAWARE
MARYLAND
D.C.
NEVADA
ILL.
IND.
OHIO
W. VA.
CALIFORNIA
UTAH
COLORADO
KANSAS
MISSOURI
KENTUCKY
VIRGINIA
N.C.
TENNESSEE
OKLAHOMA
ARK.
S.C.
ARIZONA
NEW MEXICO
MISS.
ALA.
GEORGIA
LA.
TEXAS
FLORIDA
MEXICO

24

31

32

NUMBER TRICKS

1. Pick a number, 1 through 8.
   Multiply the number by 3.
   Add 1 to the product.
   Multiply the sum by 3.
   Add 8 to the product.
   The answer is the digit in the tens place of the sum.

2. Multiply your age by 3.
   Add 6 to the answer.
   Divide the sum by 3.
   Ask for the result.
   Subtract 2 from the answer.
   The result will be your age.

3. Write the month and day of your birthday, using 1 for January, 2 for February, etc.
   Multiply the month of your birth by 5.
   Add 6.
   Multiply by 4.
   Add 9.
   Multiply by 5.
   Add the day of your birth to the answer.
   Subract 165.
   The last two numbers of the answer will be the date of your birth.
   The first number (or numbers) will be the month of your birth.

4. MULTIPLY THE NUMBER 9 BY ANY OTHER NUMBER LOWER THAN 9.
SUBTRACT THIS PRODUCT FROM 10 TIMES YOUR AGE.
THE FIRST TWO DIGITS PLUS THE LAST DIGIT GIVES YOUR AGE.

5. TAKE YOUR HOUSE NUMBER.
DOUBLE IT.
ADD 5.
MULTIPLY BY 50.
ADD YOUR AGE.
ADD 365.
SUBTRACT 615.
PLACE A POINT AS YOU WOULD IF THE ANSWER WERE DOLLARS AND CENTS.
YOUR HOUSE NUMBER IS TO THE LEFT OF THE POINT AND YOUR AGE IS TO THE RIGHT.

6. CHOOSE ANY NUMBER.
MULTIPLY IT BY 2.
ADD 1.
MULTIPLY BY 5.
ADD 3.
CROSS OUT THE ONE'S DIGIT.
THE ANSWER IS THE NUMBER CHOSEN AT THE START.

7. CHOOSE ANY 3 NUMBERS LESS THAN 10.
MAKE ALL THE 2-DIGIT NUMBERS THAT YOU CAN FORM FROM THESE 3 NUMBERS. (THERE ARE ALWAYS 6 SUCH NUMBERS.)
ADD THE 6 NUMBERS.
NOW ADD THE ORIGINAL 3 NUMBERS.
DIVIDE THE FIRST SUM BY THE SECOND SUM.
THE ANSWER IS ALWAYS 22.

8. THINK OF THE MONTH OF YOUR BIRTHDAY, USING 1 FOR JANUARY, 2 FOR FEBRUARY, ETC.
MULTIPLY BY 2.
ADD 5.
MULTIPLY BY 50.
ADD YOUR AGE.
SUBTRACT 365.
ADD 115.
YOUR AGE IS SHOWN BY THE LAST TWO DIGITS IN YOUR ANSWER.
THE MONTH OF YOUR BIRTHDAY IS SHOWN BY THE FIRST DIGIT OR DIGITS OF YOUR ANSWER.

9. WRITE FOUR DIGITS IN DESCENDING ORDER.
REVERSE THE ORDER AND SUBTRACT.
REVERSE AGAIN AND ADD.
THE RESULT IS ALWAYS 10,890.

10. WRITE DOWN YOUR AGE IN YEARS.
MULTIPLY BY 2.
SUBTRACT 3.
MULTIPLY BY 50.
ADD 39.
ADD THE AMOUNT OF CHANGE YOU HAVE IN YOUR POCKET. THE AMOUNT MUST BE LESS THAN ONE DOLLAR.
ADD 111.
THE FIRST TWO DIGITS OF THE ANSWER WILL BE YOUR AGE.
THE LAST TWO DIGITS OF THE ANSWER WILL BE THE AMOUNT OF CHANGE.

11. Pick four different numbers from 0 to 9.
Arrange them to make the largest number possible.
Now arrange them to make the smallest number possible.
Subtract the small number from the large one.
Now, take the answer and arrange those digits to make the largest number possible.
Then arrange to make the smallest number possible.
Subtract.
Repeat until you get 6174 as an answer. This number will always turn up.

12. A number or word that reads the same both ways is called a palindrome. For example 11, 22, 111, 121, 202, 123321.
Write any number.
Add the number obtained by reversing the digits.
Continue this process until the answer is a palindromic.
Start a chart.

One step palindrome	Two step palindrome	Three step palindrome
13	73	86
31	37	68
44	110	154
	011	451
	121	605
		506
		1111

What number can you find with the most steps?

13.

193	138	235	380	397	234	134	441
60	5	102	247	264	101	1	308
399	344	441	586	603	440	340	647
64	9	106	251	268	105	5	312
204	149	246	391	408	245	145	452
59	4	101	246	263	100	0	307
101	46	143	288	305	142	42	349
280	225	322	467	484	321	221	528

1. CIRCLE ANY NUMBER.
2. DRAW A VERTICAL LINE THROUGH ALL THE NUMBERS IN THE SAME COLUMN AND A HORIZONTAL LINE THROUGH ALL THE NUMBERS IN THE SAME ROW AS THE CIRCLED NUMBER.
3. CIRCLE ANY NUMBER NOT CROSSED OUT. REPEAT PREVIOUS PROCEDURE: CROSS OUT ALL NUMBERS IN SAME ROW AND COLUMN AS THE NUMBER JUST CIRCLED.
4. CONTINUE IN THIS WAY, CIRCLING ANY NUMBER NOT CROSSED OUT AND ELIMINATING ALL NUMBERS IN SAME ROW AND COLUMN.
5. WHEN ONLY ONE NUMBER REMAINS, CIRCLE IT.
6. ADD THE EIGHT CIRCLED NUMBERS.
7. PUT YOUR ANSWER HERE _____.

38

LOGIC PUZZLES

IT IS INTERESTING TO OBSERVE THAT PUZZLES OF THE PURELY LOGICAL TYPE ARE REPRESENTATIVE OF THE ENTIRE SCIENTIFIC PROCESS. AT THE BEGINNING ONE IS CONFRONTED WITH A MASS OF MORE OR LESS UNRELATED DATA. FROM THESE FACTS A FEW POSITIVE INFERENCES CAN BE SEEN IMMEDIATELY, BUT USUALLY IT IS NECESSARY TO MAKE ASSUMPTIONS TO GUIDE THE SEARCH FOR A SOLUTION. THE VALIDITY OF THESE ASSUMPTIONS MUST BE CHECKED BY TESTING FOR CONSISTENCY WITH THE ORIGINAL DATA. IF INCONSISTENCIES APPEAR, THE ASSUMPTIONS NUST BE REJECTED AND OTHERS TIRED UNTIL FINALLY A CONSISTENT SET OF CONCLUSIONS EMERGES.

THE SOLUTION OF LOGIC PUZZLES CANNOT BE REDUCED TO A FIXED PATTERN. NEVERTHELESS, THERE ARE SOME GENERAL SUGGESTIONS ON HOW TO ATTACK PUZZLES OF THIS SORT.

EXAMPLE:

JONES, SMITH, JOHNSON, AND MAYS ARE FOUR TALENTED ARTISTS, ONE A DANCER, ONE A PAINTER, ONE A SINGER, AND ONE A WRITER (NOT NECESSARILY IN THAT ORDER).

1) JONES AND JOHNSON WERE IN THE AUDIENCE THE NIGHT THE SINGER MADE HIS DEBUT ON THE CONCERT STAGE.
2) BOTH SMITH AND THE WRITER HAVE SAT FOR PORTRAITS BY THE PAINTER.
3) THE WRITER, WHOSE BIOGRAPHY OF MAYS WAS A BEST-SELLER, IS PLANNING TO WRITE A BIOGRAPHY OF JONES.
4) JONES HAS NEVER HEARD OF JOHNSON. WHAT IS EACH MAN'S ARTISTIC FIELD?

TO KEEP TRACK MENTALLY OF THESE MANY FACTS AND THE HYPOTHESES

AND CONCLUSIONS BASED UPON THEM IS CONFUSING AND DIFFICULT. THEREFORE, MAKING A CHART WHICH SHOWS ALL POSSIBILITIES IS A GOOD WAY TO START YOUR SOLUTION.

	DANCER	PAINTER	SINGER	WRITER
JONES				
SMITH				
JOHNSON				
MAYS				

NOW FROM 1), IT IS KNOWN THAT NEITHER JONES NOR JOHNSON IS THE SINGER. PLACE AN X OPPOSITE THEIR NAMES IN THE COLUMN HEADED BY THE SINGER.

FROM 2), IT IS KNOWN THAT SMITH IS NEITHER THE PAINTER NOR THE WRITER. PLACE AN X OPPOSITE SMITH'S NAME IN THE TWO COLUMNS HEADED PAINTER AND WRITER.

FROM 3), THE WRITER IS NEITHER MAYS NOR JONES. PLACE AN X OPPOSITE THE NAMES, MAYS AND JONES IN THE COLUMN HEADED WRITER.

THE CHART NOW LOOKS LIKE THE FOLLOWING:

	DANCER	PAINTER	SINGER	WRITER
JONES			X	X
SMITH		X		X
JOHNSON			X	
MAYS				X

IF A CLUE HAD STATED THAT SMITH WAS THE SINGER, A CHECK WOULD HAVE BEEN PLACED OPPOSITE SMITH'S NAME IN THE COLUMN HEADED SINGER. HOWEVER, IN THIS PROBLEM, THERE WERE NO DIRECT FACTS GIVEN. BUT AT THIS STAGE WHEN THE CHART IS EXAMINED, IT IS CLEAR THAT JOHNSON IS THE WRITER BECAUSE ALL OTHER POSSIBILITIES ARE MARKED OFF. SO PLACE A CHECK OPPOSITE JOHNSON'S NAME IN THE COLUMN HEADED WRITER AND FILL THE REMAINING SQUARES IN HIS ROW WITH X'S.

ACCORDING TO 2), JOHNSON HAS SAT FOR THE PAINTER AND IN 4), JONES DOES NOT KNOW JOHNSON. THEREFORE, JONES IS NOT THE PAINTER. PLACE AN X BY JONES' NAME IN THE COLUMN HEADED PAINTER. BY ELIMINATION ON THE CHART, JONES MUST BE THE DANCER. PLACE X'S IN THE OTHER SQUARES IN THE DANCER COLUMN.

OBSERVE THAT THE SINGER MUST BE SMITH. ADD A CHECK AND X'S IN THE CORRECT SQUARES.

FINALLY, MAYS MUST BE THE PAINTER, AND THE SOLUTION IS COMPLETE.

PROBLEMS

1) FOUR MEN, ONE OF WHOM WAS KNOWN TO HAVE COMMITTED A CERTAIN CRIME, MADE THE FOLLOWING STATEMENTS WHEN QUESTIONED BY THE POLICE:

WILLIAM: MARK DID IT.

MARK: JOHN DID IT.

LUKE: I DIDN'T DO IT.

JOHN: MARK LIED WHEN HE SAID I DID IT.

IF ONLY ONE OF THESE FOUR STATEMENTS IS TRUE, WHO WAS THE GUILTY MAN?

2) TWO CHILDREN FROM A GROUP OF 41 BOYS AND GIRLS HAD THE CHANCE TO WIN TWO TICKETS TO SIX FLAGS. KARL AND KATHY FIGURED OUT HOW TO WIN. KARL SUGGESTED THAT THE GROUP FORM A CIRCLE AND BEGIN ELIMINATING EVERY THIRD PERSON UNTIL ONLY TWO WOULD BE LEFT. THESE REMAINING TWO PERSONS WOULD THEN BE THE WINNERS. KARL WOULD SELECT THE FIRST PERSON IN THE CIRCLE FROM WHICH TO BEGIN THE COUNTING. WHERE WOULD KARL AND KATHY PLACE THEMSELVES IN ORDER TO BE CERTAIN THAT THEY WOULD WIN?

3) "GIVE ME CHANGE FOR A DOLLAR, PLEASE," SAID THE CUSTOMER. "I'M SORRY," SAID THE CASHIER, "BUT I CAN'T DO IT WITH THE COINS I HAVE HERE." "CAN YOU CHANGE A HALF DOLLAR THEN?" THE CASHIER SHOOK HER HEAD. IN FACT, SHE COULDN'T EVEN MAKE CHANGE FOR A QUARTER, DIME, OR NICKEL! "DO YOU HAVE ANY COINS AT ALL?" ASKED THE CUSTOMER. "OH, YES," SAID THE CASHIER. "I HAVE $1.15 IN COINS." EXACTLY WHAT COINS WERE IN THE CASH REGISTER?

4) A MAN GOES INTO A STORE AND SAYS TO THE CLERK: "GIVE ME AS MUCH MONEY AS I HAVE WITH ME AND I WILL SPEND $10 WITH YOU." IT IS DONE. THE MAN REPEATS THE OPERATION IN A SECOND AND A THIRD STORE, AFTER WHICH HE HAS NO MONEY LEFT. HOW MUCH DID HE START WITH?

5) TEN BLUE SOCKS AND TEN RED SOCKS ARE ALL MIXED UP IN THE DRAWER. TWENTY SOCKS ARE EXACTLY ALIKE EXCEPT THEIR COLOR. THE ROOM IS IN PITCH DARKNESS AND YOU WANT TWO MATCHING SOCKS. WHAT IS THE SMALLEST NUMBER OF SOCKS YOU MUST TAKE OUT OF THE DRAWER IN ORDER TO BE CERTAIN THAT YOU HAVE A PAIR THAT MATCH?

6) Tom wanted his father to give him an allowance of $1.00 a week, but his father refused to go higher than 50¢. After they argued for a while, Tom said: "Tell you what, Dad. Today is the first of April. You give me a penny today. Tomorrow, give me two pennies. The day after tomorrow, give me four pennies. Each day, give me twice as many pennies as you did the day before." "For how long?" asked Dad. "Just for the month of April," said Tom. "Then I won't ask you for any more money for the rest of my life." "Okay," said Dad. Which of the following figures do you think comes closest to the amount of money that Dad will have to pay?

$1	$10	$100
$1,000	$10,000	$100,000
$1,000,000	$10,000,000	$100,000,000

7) A 158 pound farmer's daughter was going to the fair with a prize 39 pound fox, 19 pound goose, and a 40 pound sack of corn. On her way she came to a wide stream which she had to cross. It was too wide to swim across and there were no bridges. The only available means of crossing was a water-soaked rowboat. On it was this sign:

"Beware! Do not load with more than 200 pounds."

Now, if she took over the fox, the goose would eat the corn. If she took over the corn, the fox would eat the goose. This would also happen if the two were left on the other side by themselves. How can the farmer's daughter solve her problem?

8) A MAN BOUGHT A COW FOR $90.00. HE LATER SOLD IT FOR $100.00 STILL LATER, HE BOUGHT THE SAME COW BACK FOR $80.00. DID HE LOSE MONEY OR MAKE MONEY?

9) MARY HAS AN EIGHT GALLON TANK OF ORANGE JUICE. SHE WANTS TO DIVIDE IT EQUALLY BETWEEN TWO PEOPLE, BUT SHE HAS ONLY AN EMPTY FIVE GALLON TANK AND A THREE GALLON TANK TO MEASURE WITH. HOW DOES SHE SOLVE THIS PROBLEM?

10) A MONKEY IS AT THE BOTTOM OF A 30-FOOT WELL. EVERY DAY HE CLIMBS UP THREE FEET AND SLIDES BACK TWO FEET. WHEN DOES HE REACH THE TOP?

11) THREE BRIDEGROOMS, TRAVELING WITH THEIR BRIDES, CAME TO A RIVER WITH NO BRIDGE. THE ONE AVAILABLE BOAT WOULD ACCOMMODATE ONLY TWO PEOPLE. SINCE THE HUSBANDS WERE VERY JEALOUS, NO WOMAN COULD BE WITH A MAN UNLESS HER OWN HUSBAND WAS PRESENT. HOW CAN THEY GET ACROSS THE RIVER USING THE ONE BOAT.

12) IF A CATERPILLAR CRAWLS UP A UTILITY POLE THREE FEET EACH DAY, AND SLIDES DOWN TWO FEET EACH NIGHT, HOW MANY DAYS WILL IT TAKE HIM TO REACH THE TOP OF THE POLE IF THE POLE IS TEN FEET TALL?

13) JUAN, JUANITA, MARIE, AND RAMON ARE 10, 12, 13, AND 14 YEARS OLD.
1. MARIE IS OLDER THAN RAMON AND YOUNGER THAN JUAN.
2. JUANITA IS YOUNGER THAN MARIE AND OLDER THAN RAMON.
WHAT IS EACH PERSON'S AGE?

14) Ms. Arteaga, Ms. Boyd, and Ms. Cameron all went to the store on the same day. One of them purchased an 8 oz. box of Superduds soap for 69¢; one of them purchased a 16 oz. box of the same soap for $1.30; and the other one purchased a 32 oz. box of soap for $2.80.

1. Ms. Cameron always was the best comparative shopper and bought the size that would give her the most for her money.
2. The 69¢ box of soap was sold to the person with the least number of people in her home.
3. Ms. Arteaga's husband was the brother of one of the women.
4. One of the women was not married and lived alone.

Which woman bought which box of soap?

15) Al, Ben, and Charles each entered a dog in the dog show. Their dogs, which took the top three prizes, were a boxer, a collie, and a dachshund. Find each person's dog and the prize it took.

1. This was the first show for Al's dog.
2. Ben's dog was not the collie.
3. Last year, the boxer won a higher prize than the collie.
4. Al's dog won a higher prize than the collie.
5. Ben's dog won a higher pirze than Al's dog.

16) The favorite sports of Barbara, Howell, Irene, and Tona are baseball, hiking, ice-skating, and tennis.

1. No person's name begins with the same letter as his favorite sport.
2. Tona and Howell don't like team sports.
3. Howell and Barbara don't like cold weather sports.

What is each person's favorite sport?

17) George, Lewis, and Oscar live in Georgia, Louisiana, and Ohio. One drinks grape juice, one drinks lemonade, and one drinks orange juice.

1. Nobody lives in a state or drinks something which starts with the same letter as his name starts with.
2. The state where each person lives starts with a different letter than his drink.
3. George doesn't live in Louisiana.

Where does each person live? What does each person drink?

18. A green house, a pink house, a white house, and a yellow house are all in a row.

1. The yellow house is not first.
2. The pink house is between the green house and the white house.
3. The green house is between the yellow house and the pink house.

What is the position of each house?

19) Dewey, Arthur, and William are professionals. They like to keep their offices at different temperatures. They prefer temperatures of 22° C, 23° C, and 24° C but not necessarily in that order. Find out which temperature each professional keeps his office.

1. William likes his office warmer than Dewey.
2. The blonde-haired professional keeps his office the warmest.
3. William has black hair.

20) Each one of three boys, Frank, Ramiro, and Harold, had 50¢. One has two quarters. One has five dimes. The other has one quarter, two dimes, and one nickel.

1. The one who has four coins is Ramiro's best friend and is the youngest of the three.
2. Harold is older than the one who has five dimes.

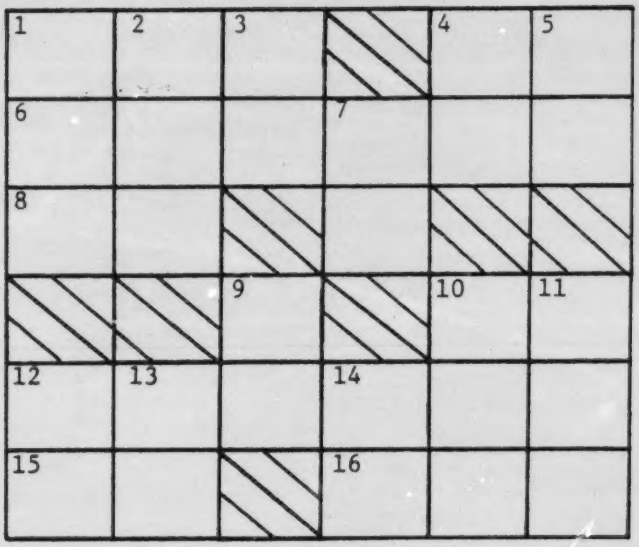
Which boy has which set of coins?

21) 1. Hamilton's portrait is on the $10.

2. Grant, Franklin, and Washington together make $151.
3. Chase and Madison together make $15,000.
4. McKinley and Jackson together make $520.
5. Madison, Grant, and Cleveland together make $6050.
6. Franklin, Jackson and Lincoln together make $125.

Whose pictures are on the following bills?

$1	_____	$100	_____
$5	_____	$500	_____
$10	_____	$1000	_____
$20	_____	$5000	_____
$50	_____	$10,000	_____



ACROSS

1) $29 + 33 + 15 + 24$

4) 3×8

6) 1 LESS THAN A MILLION

8) $4704 \div 12 = __ __ 2$

10) $14\frac{1}{2} + 12\frac{1}{3} + 5\frac{1}{6}$

12) $(512 \times 609) + 453$

15) 1, 3, 7, 15, 31, ___, ---

16) AREA: 4 BY 103

DOWN

1) CXCIII

2) $7265 - 4166 = 3 __ __ __$

3) $532 \div 28$

4) $28 < __ __ < 30$

5) $\frac{1}{2}$ OF 100 MINUS 1

7) $2425 \div 25$

9) $\frac{84}{91} = \frac{?}{13}$

10) 19×19

11) $7844 \div 37$

12) $7\frac{1}{5} = \frac{?}{5}$

13) $? \times 24 = 312$

14) $\frac{1}{6} + \frac{1}{8} = \frac{7}{?}$

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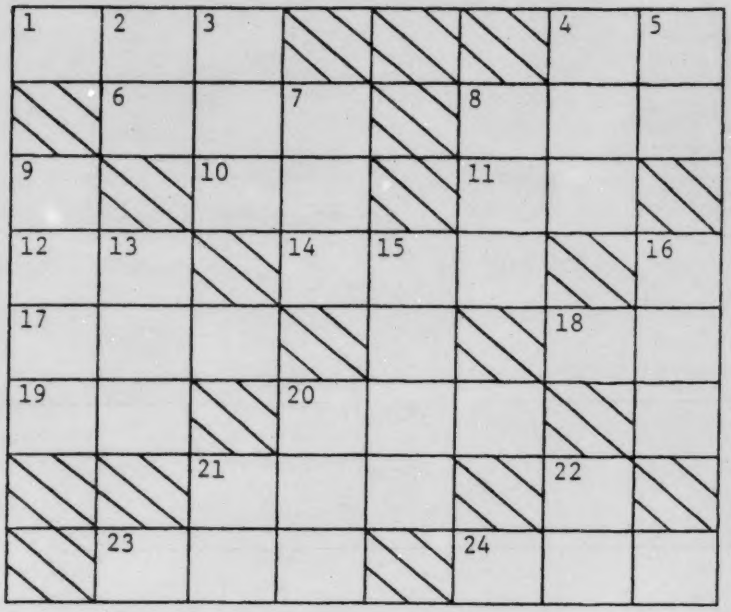

ACROSS

1) 43380 MIN. = ? HR.

4) $38 < ? < 40$

6) 189 + 143 + 131

8) 8 HR. = ? MIN.

10) 17 YD. = ? FT.

11) $\frac{1}{2} = \frac{?}{72}$

12) 3 + 9 + 7 + 8 + 4

14) 109 x 7

17) 25496 - 24968

18) RIGHT ANGLE HAS ? DEGREES.

19) $25 < ? < 27$

20) 15566 ÷ 43

21) 1941 - 989

23) 123 + 198 + 84

24) 14603 ÷ 17

DOWN

2) 1 DAY = ? HR.

3) NO. OF DAYS IN A REGULAR YEAR.

4) $385 < ? < 387$

5) AN OBTUSE ANGLE $>$? DEGREES

7) 2536 ÷ 8

8) PERIMETER OF TRIANGLE WITH SIDES 56, 102, 275

9) 98 x 24

13) PERIMETER OF RECTANGLE WITH TWO SIDES 36 AND 27

15) 237 x 26

16) 5672 ÷ 8

20) 71 x 5

21) AN ACUTE ANGLE $<$? DEGREES

22) $\frac{3}{5} = \frac{39}{?}$

LETTERS MAKE WORDS AND WORDS MAKE CENTS!

A = 1¢	B = 2¢	C = 3¢	D = 4¢	E = 5¢	F = 6¢
G = 7¢	H = 8¢	I = 9¢	J = 10¢	K = 11¢	L = 12¢
M = 13¢	N = 14¢	O = 15¢	P = 16¢	Q = 17¢	R = 18¢
S = 19¢	T = 20¢	U = 21¢	V = 22¢	W = 23¢	X = 24¢
Y = 25¢	Z = 26¢				

1. DIRECTIONS:

   EACH LETTER HAS A PRICE. FIGURE THE VALUE OF THE WORD.

   WHICH IS WORTH THE MOST? A NICKLE, A QUARTER, OR A DOLLAR?

2. DIRECTIONS:

   PLAY WITH TWO TO FOUR PEOPLE. EACH PLAYER HAS A PIECE OF PAPER AND A PENCIL. FOR EACH OF THE CLUES LISTED BELOW, SELECT A WORD AND WRITE IT ON YOUR PAPER. USING THE LETTER PRICE CHART, CALCULATE THE VALUE OF YOUR WORDS. TOTAL THE VALUES. THE PERSON WITH THE HIGHEST TOTAL WINS.

   1) A COLOR
   2) A FOOD
   3) A CITY
   4) A MONTH
   5) A PIECE OF CLOTHING
   6) A PET
   7) A DRINK
   8) A COUNTRY
   9) A PIECE OF FURNITURE
   10) A FORM OF TRANSPORTATION

SUPPRESSED DIGITS

CAN YOU RESTORE THE UNKNOWN DIGITS REPRESENTED BY STARS?

ADDITION:

1)
```
    9
    8
  9 * *
-------
1 0 0 0
```

2)
```
  * 0 0 *
  4 0 * 1
  3 * 6 0
---------
  9 9 9 0
```

3)
```
  1 2 *
    * 7
  * 4 3
-------
  6 3 2
```

SUBTRACTION:

4)
```
  * 0 0 0 *
    9 9 9 9
-----------
          9
```

5)
```
  9 * 3 *
  * 2 * 3
---------
  2 0 7 2
```

6)
```
  * 7 * 2 *
  7 * 5 * 8
-----------
  1 7 8 0 6
```

MULTIPLICATION:

7)
```
  * 4 * 8 *
      5 0 5
-----------
  7 2 9 1 0
7 2 9 1 0 0
-----------
* * * * * * *
```

8)
```
  2 3 *
  7 * 3
-------
  7 1 *
1 * 7 3 0
---------
* 6 * * 1 *
```

9)
```
    5 *
    * 6
-------
  3 1 *
* * 7
-------
5 0 * *
```

DIVISION:

10)
```
            * * 3
        ---------
5 * 9 | 1 7 5 4 3 *
        1 7 * *
        -------
            * 7 3 7
            * 7 3 7
            -------
```

11)
```
              * 7
          -------
* 6 3 | 9 * 7 *
       * 6 3
       -------
       * 9 * 1
       3 * * *
       -------
```

12)
```
              * * 4 * *
          -------------
* 5 | 8 * 4 7 * 5
      * 5
      -------
        3 * *
        * * 0
        -----
            * * 5
            * 6 *
            -----
```

VICTORIAN ARITHMETIC

SOME INTERESTING PROBLEMS ARE PRINTED FROM THE NATIONAL ARITHMETIC, BY BENJAMIN GREENLEAF, PUBLISHED IN 1842. QUEEN VICTORIA RULED FROM 1837 TO 1901 AS QUEEN OF ENGLAND. BECAUSE OF HER INFLUENCE DURING THIS TIME, IT IS KNOWN AS THE VICTORIAN ERA.

1. REQUIRED THE NUMBER OF INHABITANTS IN THE NEW ENGLAND STATES, THERE BEING IN MAINE 501,793; IN NEW HAMPSHIRE 284,574; IN MASSACHUSETTS 737,699; IN RHODE ISLAND 108,830; IN CONNECTICUT 309,978; IN VERMONT 291,948.

KEEP IN MIND THAT TEXAS WAS A NATION BETWEEN 1836 AND 1845. THEREFORE, IT WAS NOT A STATE IN 1842. ALSO, COMPARE THE POPULATION OF FORT WORTH NOW WHICH IS ABOUT 390,000 TO DIFFERENT STATES THEN.

2. HOW MANY INHABITANTS IN THE WESTERN STATES, THERE BEING IN TENNESSEE 829,210; IN KENTUCKY 779,828; IN OHIO 1,519,467; IN INDIANA 685,866; IN ILLINOIS 476,183; IN MISSOURI 383,702; IN ARKANSAS 97,574; IN MICHIGAN 212,267?

3. VULGAR FRACTIONS ARE EXPRESSED BY TWO NUMBERS, CALLED THE NUMERATOR AND DENOMINATOR; THE FORMER ABOVE, AND THE LATTER BELOW A LINE, $\frac{7}{11}$ WHAT DO WE CALL THESE FRACTIONS TODAY?

4. IF A SPORTSMAN SPEND $\frac{1}{3}$ OF HIS TIME IN SMOKING, $\frac{1}{4}$ IN "GUNNING," 2 HOURS PER DAY IN LOAFING, AND 6 HOURS IN EATING, DRINKING, AND SLEEPING, HOW MUCH REMAINS FOR USEFUL PURPOSES?

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5. If a lady spend $\frac{1}{3}$ of her time in sleep, $\frac{1}{12}$ in making calls, $\frac{1}{8}$ at her toilet, $\frac{1}{6}$ in reading novels, and 2 hours each day receiving visits, how large a portion of her time will remain for improving her mind and domestic employments?

6. What part of an orange is a $\frac{1}{2}$ of a half?

7. What part of an apple is a $\frac{1}{4}$ of a half?

8. Borrowed of James Day $150 for six months; afterwards I lent him $100; how long shall he keep it to indemnify him for the sum he lent me?

9. What number is that, to which, if $\frac{1}{8}$ be added, the sum will be $7\frac{1}{2}$?

10. How many cubic feet are there in a cube whose sides are 8 inches?

11. A loafer shot at a flock of pigeons on a tree, and killed 24, which was $\frac{3}{7}$ of the number. How many pigeons will remain on the tree?

12. Sold a quantity of depreciated money for $81, which was $\frac{9}{11}$ of its nominal value; what was the sum sold?

13. Bought a horse for $72, which was $\frac{8}{9}$ of his real value; what did I gain?

14. Samuel Page sold a pair of oxen for $48, which was $\frac{6}{7}$ of their cost. What did he lose?

15. D. WEBSTER BOUGHT $\frac{3}{7}$ OF A SAW MILL, FOR WHICH HE PAID $300. WHAT WAS THE VALUE OF THE WHOLE MILL?

16. A LADY GAVE HER DAUGHTER $10 TO GO A "SHOPPING;" HAVING PURCHASED 2 YD. OF SILK, AT $1.27 PER YD., A BONNET FOR $3.75, 3 PAIRS OF GLOVES AT $0.19 A PAIR, AND TWO FANS AT $0.37 EACH, SHE RETURNED THE REMAINDER OF THE MONEY TO HER MOTHER; WHAT WAS THE SUM?

17. FROM $10, $2\frac{1}{8}$ WAS GIVEN TO BENJAMIN, $3\frac{1}{4}$ TO LYDIA, $1\frac{1}{2}$ TO EMILY, AND THE REMAINDER TO BETSEY; WHAT DID SHE RECEIVE?

18. SOUND, UNINTERRUPTED, WILL PASS 1142 FEET IN ONE SECOND; HOW LONG WILL IT BE IN PASSING FROM BOSTON TO LONDON, THE DISTANCE BEING ABOUT 3000 MILES? THERE ARE 5280 FEET IN ONE MILE.

19. THE TIME WHICH ELAPSED BETWEEN SEEING THE FLASH OF A GUN, AND HEARING ITS REPORT, WAS 10 SECONDS; WHAT WAS THE DISTANCE?

20. A YOUNG MAN LOST $\frac{1}{4}$ OF HIS CAPITAL IN SPECULATION; HE AFTERWARDS GAINED $500; HIS CAPITAL THEN WAS $1250; WHAT WAS THE SUM LOST?

GRAPHING

ORDERED NUMBER PAIRS ARE USED TO DESCRIBE POINTS IN A PLANE. START WITH TWO SIGNED (+,-) NUMBER LINES, CALLED COORDINATE AXES, DRAWN AT RIGHT ANGLES TO EACH OTHER. THE HORIZONTAL (EAST-WEST) LINE IS CALLED THE X-AXIS. THE VERTICAL (NORTH-SOUTH) LINE IS CALLED THE Y-AXIS. IN A COORDINATE PLANE, THE POINT 0 AT WHICH THE TWO AXES INTERSECT (CROSS) IS CALLED THE ORIGIN.

THE X-AXIS AND THE Y-AXIS DIVIDE THE PLANE INTO FOUR REGIONS CALLED QUADRANTS. THESE QUADRANTS ARE NUMBERED I, II, III, AND IV IN A COUNTERCLOCKWISE ORDER.

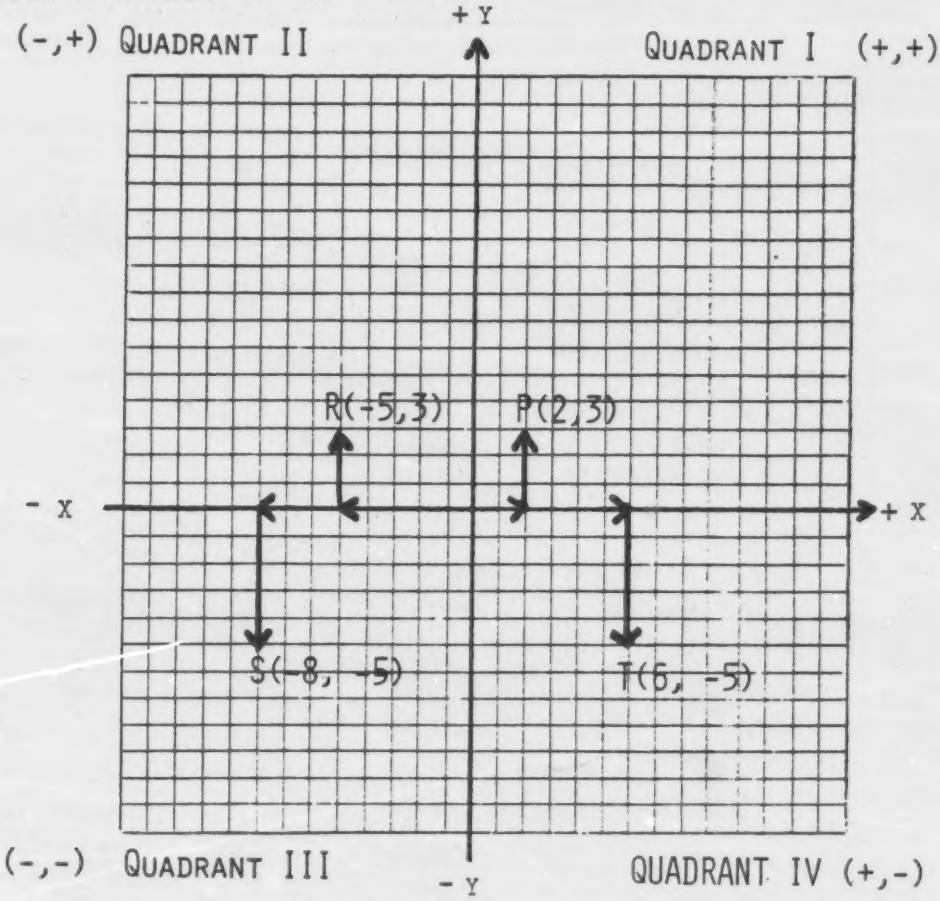
IT IS UNDERSTOOD IN ALL NUMBERS PAIRS THAT THE FIRST NUMBER ALWAYS REPRESENTS A DISTANCE ALONG THE X-AXIS; THE SECOND NUMBER ALWAYS REPRESENTS A DISTANCE ALONG THE Y-AXIS. FOR THIS REASON, IT IS NECESSARY NOT TO INTERCHANGE THE NUMBERS IN AN ORDERED PAIR.

DISTANCES MEASURED TO THE RIGHT OF Y-AXIS, ALONG THE X-AXIS OR ALONG A LINE PARALLEL TO THE X-AXIS, ARE CONSIDERED TO BE POSITIVE (+); DISTANCES MEASURED TO THE LEFT OF THE Y-AXIS ARE CONSIDERED TO BE NEGATIVE (-). DISTANCES MEASURED UPWARD FROM THE X-AXIS, ALONG THE Y-AXIS OR ALONG A LINE PARALLEL TO THE Y-AXIS, ARE CONSIDERED TO BE POSITIVE (+); DISTANCES MEASURED DOWNWARD FROM THE X-AXIS ARE CONSIDERED TO BE NEGATIVE (-). ALL NUMBERS ARE SIGNED NUMBERS. IF THERE IS NO SIGN IN FRONT OF THE NUMBER, THEN IT IS UNDERSTOOD TO BE POSITIVE.

THE DISTANCE OF A POINT FROM THE Y-AXIS, MEASURED EITHER ALONG THE X-AXIS OR ALONG A LINE PARALLEL TO IT, IS CALLED THE X-COORDINATE OR ABSCISSA. THE DISTANCE OF A POINT FROM THE

X-AXIS, MEASURED EITHER ALONG THE Y-AXIS OR ALONG A LINE PARALLEL TO IT, IS CALLED THE Y-COORDINATE OR ORDINATE. THE TWO NUMBERS WHICH ARE ASSOCIATED WITH ANY PARTICULAR POINT, THE ABSCISSA AND ORDINATE OF THE POINT, ARE CALLED THE COORDINATES OF THE POINT.

TO GRAPH THE POINT P, REPRESENTED BY THE ORDERED PAIR (2,3), START AT THE ORIGIN AND MOVE 2 UNITS (SQUARES) TO THE RIGHT ALONG THE X-AXIS, THEN MOVE 3 UNITS (SQUARES) UPWARD IN A DIRECTION PARALLEL TO THE Y-AXIS.



TO GRAPH R, REPRESENTED BY THE ORDERED PAIR (-5,3), START AT THE ORIGIN AND MOVE 5 UNITS TO THE LEFT ALONG THE X-AXIS, THEN MOVE 3 UNITS UPWARD IN A DIRECTION PARALLEL TO THE Y-AXIS.

To graph point S, represented by the ordered pair (-8, -5), start at the origin and move 8 units to the left along the x-axis, then move 5 units downward in a direction parallel to the y-axis.

To graph point T, represented by the ordered pair (6,-5), start at the origin and move 6 units to the right along the x-axis, then move 5 units downward in a direction parallel to the y-axis.

To graph the exercises, plot the first point, plot the second point, and then connect the two as you would in a dot to dot drawing. In the exercise start at the top of the first column of the first problem and read down. Then go to the second column of the first problem and read down. Continue in the same manner for all problems. Continue ploting the points but be careful to connect the points as you go. If you wait to connect them until you finish graphing all of the points, you may connect the wrong points. If you plot the points and connect them correctly, each graph will form a picture.

GRAPHING

1) (2, -9)
(2, -3)
(-2, -3)
(-2, -9)
(7, -9)
(7, 8)
(9, 6)
(-4, 11)
(-4, 12)
(-6, 12)
(-6, 9)

(0, 15)
(9, 6)
(7, 8)
(7, -9)
(-2, -9)
START NEW LINE
(-2, 2)
(-1, 2)
(-1, 6)
(-5, 6)
(-5, 2)
(-2, 2)

(-2, 6)
(-4, 6)
(-4, 2)
START NEW LINE
(2, 2)
(1, 2)
(1, 6)
(5, 6)
(5, 2)
(2, 2)
(2, 6)
(4, 6)
(4, 2)

2) (2, 14)
(-2, 7)
(2, 7)
(-2, 14)
(9, -5)
(3, -5)
(2, -3)
(3, -4)
(4, -3)

(5, -4)
(6, -3)
(7, -4)
(8, -3)
(9, -5)
(-3, -5)
(0, 1)
(3, -5)
(-9, -5)

(-8, -3)
(-7, -4)
(-6, -3)
(-5, -4)
(-4, -3)
(-3, -4)
(-2, -3)
(-3, -5)
(-9, -5)
(-2, 7)

3) (-1, 5)
(-6, 5)
(-6, -6)
(7, -6)
(7, 5)
(1, 5)
(1, -6)
(-1, -6)
(-1, 5)
(1, 5)

(7, 6)
(8, 4)
(7, 3)
(8, 4)
(7, 6)
(8, 7)
(5, 8)
(1, 5)
(0, 6)
(5, 11)

(6, 9)
(1, 5)
(0, 6)
(1, 11)
(4, 10)
(0, 6)
(-1, 11)
(-4, 9)
(0, 6)
(-5, 9)

(-7, 6)
(-6, 6)
(-8, 3)
(-6, 2)
(-8, 3)
(-6, 6)
(-1, 5)
(0, 6)

4) (1,11)
(0,12)
(1, 13)
(1, 9)
(8, 1)
(9, 1)
(1, 1)

(1,10)
(-11, -1)
(1, 1)
(1, 0)
(4, 0)
(4, -1)
(8, -1)

(6, -4)
(-6, -4)
(-12, -1)
(4, -1)
(4, 0)
(-2, 0)
(-2, -1)

5) (3, -1)
(6, 1)
(6, 3)
(4, 3)
(4, 4)
(7, 4)

(7, 1)
(3, -2)
(6, -2)
(4, -4)
(-4, -4)
(-6, -2)
(3, -2)

(3, -1)
(4, 3)
(4, 4)
(-4, 4)
(-3, -1)
(-3, -2)

6) (2, -3)
(9, 0)
(9, -6)
(-9, -6)

(0, -2)
(9, -6)
(9, 0)
(0, 3)

(-9, 0)
(-2, -3)
(-9, -6)
(-9, 0)

7) (11, 1)
(9, 5)
(9, 10)
(10, 9)
(9, 8)
(9, 5)
(-9, 5)

(-9, 8)
(-8, 9)
(-9, 10)
(-9, 5)
(-11, 1)
(11, 1)
(10, 1)
(12, -7)

(3, -7)
(3, -3)
(0, -2)
(-3, -3)
(-3, -7)
(3, -7)
(-12, -7)
(-10, 1)

8) (4, -5)
(5, -5)
(5, 4)
(-5, 4)
(-5, -5)
(4, -5)
(5, 0)
(0, 4)
(-5, 0)
(-4, -5)
START NEW LINE
(7, -6)
(11, -6)
(11, 5)
(-11, 5)
(-11, -6)
(-7, -6)
(-7, 5)
(7, 5)
(7, -6)
(-7, -6)

GEOMETRIC SHAPES

DEFINITIONS

CUBE IS A SOLID BOUNDED BY SIX PLANES, WITH ITS TWELVE EDGES ALL EQUAL AND ITS FACE ANGLES ALL RIGHT ANGLES (DICE).

PARALLELOGRAM IS A QUADRILATERAL WITH ITS OPPOSITE SIDES PARALLEL.

POLYGON IS A SIMPLE CLOSED CURVE CONSISTING OF THREE OR MORE SEGMENTS, EACH PAIR OF ADJACENT SEGMENTS HAVING A COMMON END POINT.

PRISM IS A SOLID WITH TWO CONGRUENT, PARALLEL FACES WHICH ARE POLYGONS AND WITH THE REMAINING FACES PARALLELOGRAMS.

PRISM WITH TRAPEZOIDAL BASES IS A PRISM WHOSE TWO CONGRUENT, PARALLEL FACES ARE TRAPEZOIDS.

PRISM WITH TRIANGULAR BASES IS A PRISM WHOSE TWO CONGRUENT, PARALLEL FACES ARE TRIANGLES.

QUADRILATERAL IS A POLYGON WITH FOUR SIDES.

RECTANGLE IS A QUADRILATERAL WHOSE ANGLES ARE ALL RIGHT ANGLES. ALL SQUARES ARE RECTANGLES. ALL RECTANGLES ARE NOT SQUARES.

RHOMBUS IS A QUADRILATERAL WITH FOUR EQUAL SIDES.

SQUARE IS A QUADRILATERAL WITH EQUAL SIDES AND EQUAL ANGLES.

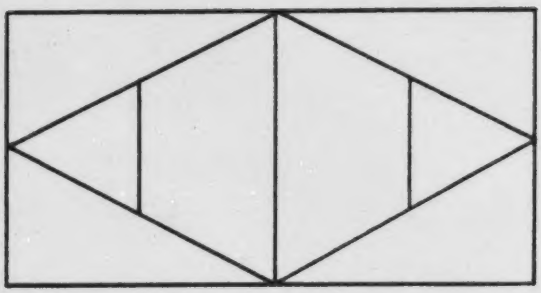
TRAPEZOID IS A QUADRILATERAL WITH EXACTLY TWO PARALLEL SIDES.

TRIANGLE IS A POLYGON WITH THREE SIDES.

GEOMETRIC SHAPES

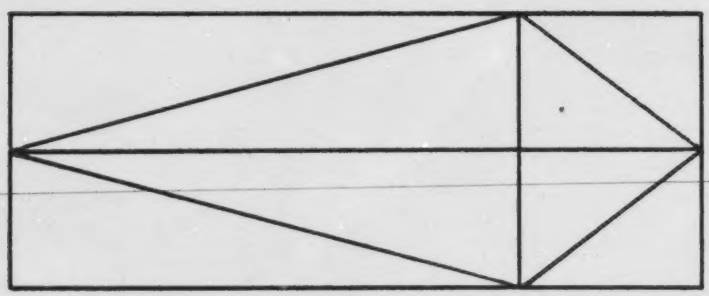
1) How many triangles?

How many rectangles?



2) How many triangles?

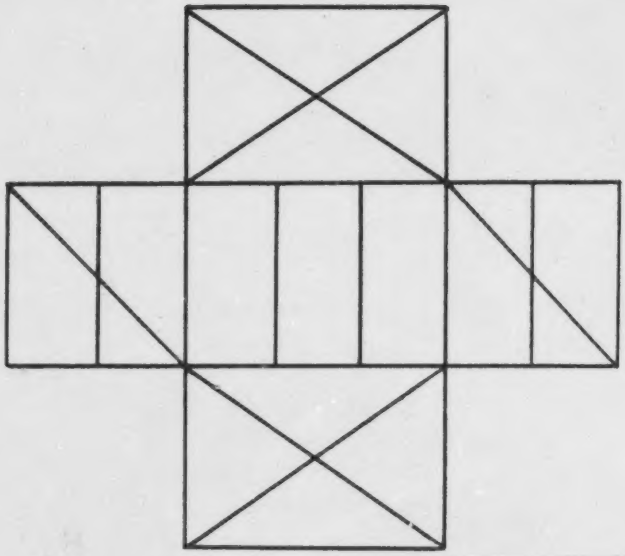
How many rectangles?



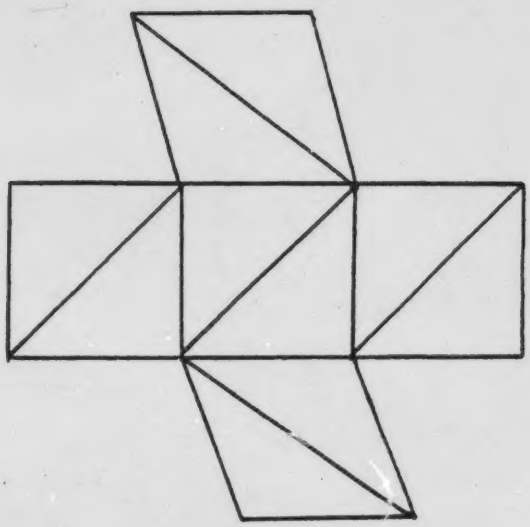
3) How many triangles?

How many squares?

How many rectangles?



GEOMETRIC SHAPES



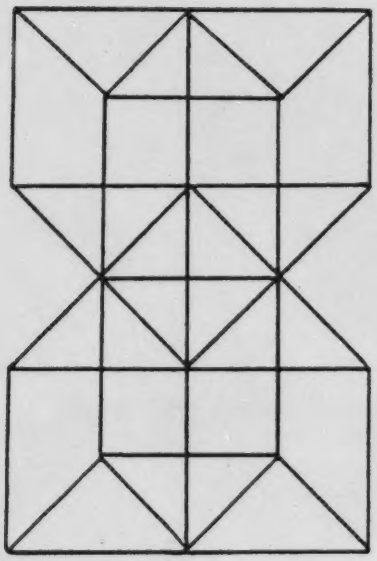
_____ How many squares?

_____ How many rectangles?

_____ How many triangles?

_____ How many quadrilaterals?

GEOMETRIC SHAPES



_____ HOW MANY SQUARES?

_____ HOW MANY RECTANGLES?

_____ HOW MANY TRIANGLES?

_____ HOW MANY CUBES?

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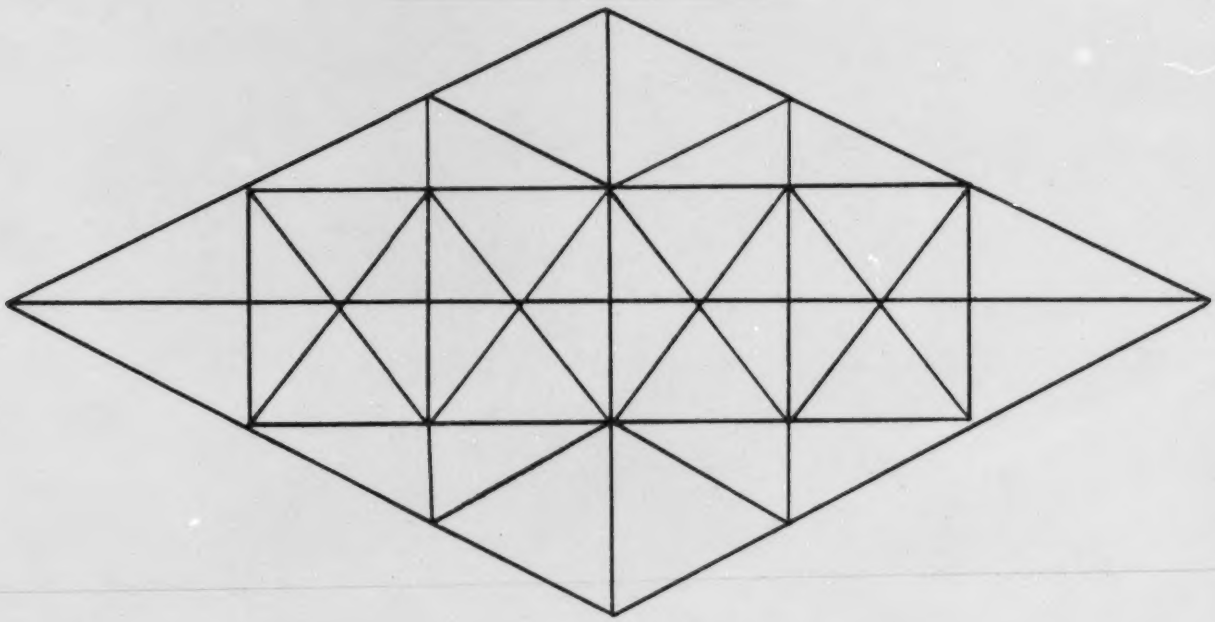
GEOMETRIC SHAPES



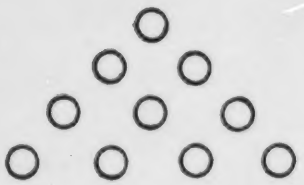
_____ How many rectangles?

_____ How many triangles?

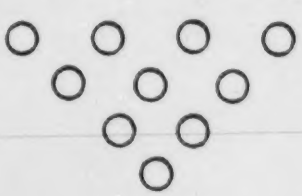
_____ How many quadrilaterals?

SPACIAL PERCEPTION

Arrange coins, buttons, paper clips, or other objects in this pattern.

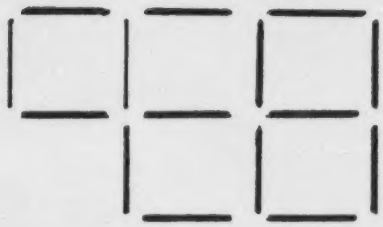


Change to this pattern by moving only three.



SPATIAL PERCEPTION

1.

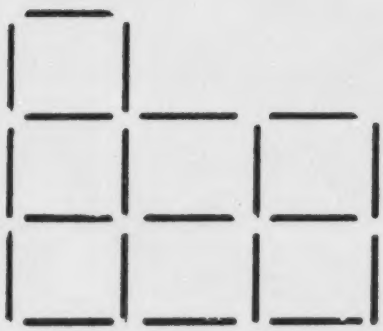


REMOVE 3 STICKS TO MAKE THREE SQUARES.

2.

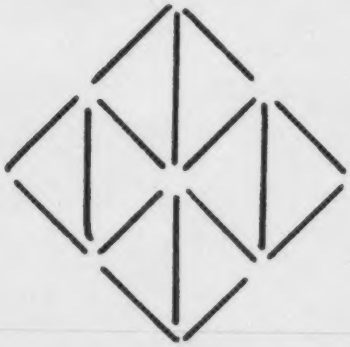
REMOVE 4 STICKS TO MAKE TWO SQUARES.

3.



REMOVE 3 STICKS TO MAKE FIVE SQUARES.

4.



REMOVE 7 STICKS TO MAKE TWO TRIANGLES

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SPATIAL PERCEPTION

1.

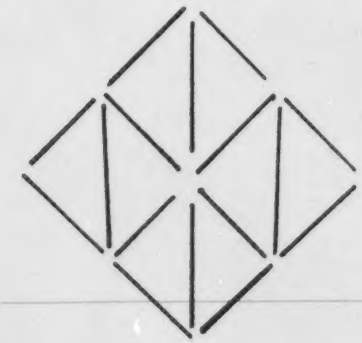
REMOVE 1 STICK, REARRANGE AS NECESSARY TO STILL HAVE SIX "PENS."

2.

REARRANGE 2 STICKS TO MAKE FOUR SQUARES.

3.

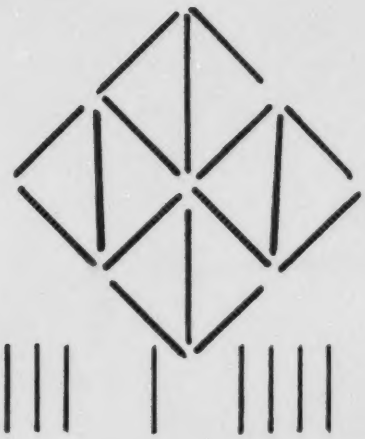
ADD 7 STICKS TO DIVIDE THE LARGE SQUARE INTO THREE PARTS OF EQUAL AREA.

4. 

REMOVE 9 STICKS TO MAKE TWO TRIANGLES.

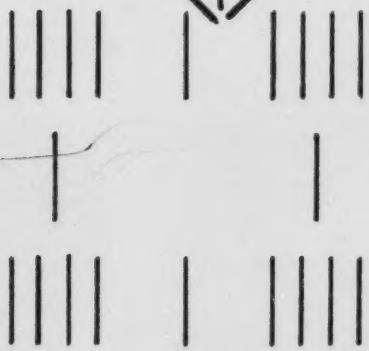
SPATIAL PERCEPTION

1.



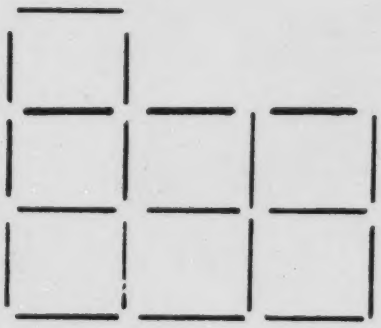
REMOVE 8 STICKS TO MAKE TWO TRIANGLES.

2.



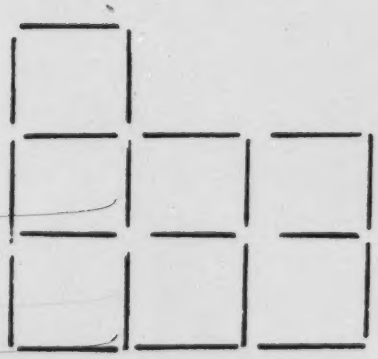
MOVE 4 STICKS THEN ADD 4 STICKS SO THAT THE TOTAL NUMBER OF STICKS ON EACH OF THE FOUR SIDES STAYS 9.

3.



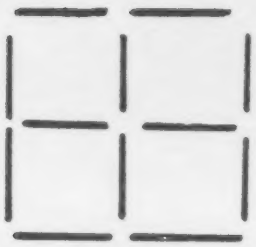
REARRANGE 3 STICKS TO MAKE FIVE SQUARES.

4.



REARRANGE 3 STICKS TO MAKE SIX SQUARES.

SPATIAL PERCEPTION

1. 

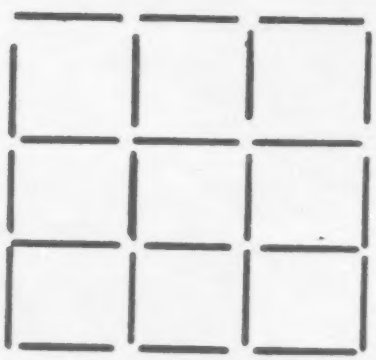
CHANGE THE NUMBER OF SQUARES FROM FOUR TO THREE BY CHANGING THE POSITION OF 3 STICKS.

2.

REMOVE 6 STICKS TO MAKE TWO TRIANGLES.

3.

REMOVE 1 STICK, REARRANGE AS NECESSARY TO STILL HAVE FOUR SQUARES.

4. 

REMOVE 8 STICKS TO MAKE THREE SQUARES.

WORD PROBLEMS

Problem Solving

Remember! Take time to read the problem with care to decide:

1. What is given.
2. What is to be found.
3. What is needed to solve the problem.
4. How you plan to solve the problem.

Examples:

1. Exact Information (just right amount)

If David needs one 12 exposure roll, two 24 exposure rolls, and three 20 exposure rolls, how much will it cost him? How many pictures can he take from the rolls that he buys?

One roll of 12 exposure $ 1.79

two rolls of 24 exposures

2 x $3.29 =6.58

three rolls of 20 exposures

3 x $2.79 =8.37

$16.74

It will cost David $16.74.

He will be able to take 120 pictures from the six rolls of film.

one roll of 12 exposure 12

two rolls of 24 exposure 2 x 24 48

three rolls of 20 exposure 3 x 20 60

120

70

2. Extraneous Information (too much)

Sue has $2.25. She found $5.00 on the way to the store. She spent $1.50 for a notebook, 25¢ for an eraser, 50¢ for a pen. How much money did she spend?

She spent:

NOTEBOOOK	$ 1.50
ERASER	.25
PEN	.50
	$ 2.25

Knowing that Sue had $2.25 and that she found $5.00 is unneeded information for solving the problem. Be sure to shift all the known facts to find only what is needed.

3. Insufficient Information (not enough)

A. Bill has 17 marbles in 3 bags. He found 2 more bags of marbles. How many marbles does he have now?

This problem cannot be solved until it is determined how many marbles are in each of the 2 bags that Bill found. If you find a problem with insufficient information, state what piece of information is needed to solve the problem.

B. Mary is 5 years older than her brother. How old is Mary?

It is necessary to know how old Mary's brother is before you can determine Mary's age.

4. No Numerical Solution

William is going to buy a new car. William's father said that he must know all the expenses connected with owning a car before William has permission to buy the car. Besides the car payment, list some additional expenses for owning a car.

This type of problem does not have a numerical solution. However, it is a practical problem which needs solving.

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ADVERTISING PUZZLES

(DISREGARD SALES TAX)

1) MRS. WILSON IS BUYING MEAT TO PUT IN HER FREEZER. SHE WANTS 50 LBS. OF GROUND BEEF, 20 LBS. OF CHUCK ROAST, 6 LBS. OF STEW MEAT, AND 10 LBS. OF CUBE STEAK. HOW MUCH WILL IT COST?

2) JANE IS HAVING A PARTY FOR 10 GIRLS. HOW MANY FRANKS WILL SHE BUY?

3) MR. GREEN IS HAVING A COOKOUT FOR A BOY SCOUT GROUP. HE FIGURES EACH BOY WILL EAT 4 HOT DOGS AND THERE WILL BE 17 BOYS AT THE PARTY. . HE IS GOING TO EAT 2 HOT DOGS. HOW MANY FRANKS WILL HE NEED?

4) HOW MUCH WILL MR. GREEN HAVE TO PAY FOR THE FRANKS? REMEMBER THAT PACKAGES OF FRANKS CANNOT BE SPLIT.

5) WILL IT BE CHEAPER TO BUY 5 LBS. OF FAMILY PACK PORK CHOPS OR 4 LBS. OF CENTER CUT PORK CHOPS?

ADVERTISING PUZZLES

(DISREGARD SALES TAX)

1) HOW MUCH WOULD IT COST MR. BROWN TO BUY 23 SQUARE YARDS OF SAXONY NYLON?

2) HOW MUCH WOULD IT COST MR. BROWN TO BUY 23 SQUARE YARDS OF SCULPTURED NYLON?

3) HOW MUCH MORE WOULD HE HAVE TO PAY FOR SAXONY NYLON THAN THE SCULPTURED NYLON TO CARPET HIS DEN?

4) MRS. BROWN DECIDED THAT SHE WANTED TO RECARPET HER BEDROOM AT THE SAME TIME. THEY WILL NEED 16 SQUARE YARDS OF THE 50 OZ. SAXONY. HOW MUCH WILL IT COST?

5) SINCE MR. BROWN HAS $500 TO SPEND FOR CARPETING BOTH ROOMS, WILL HE CHOOSE COLORFAIR OR SUNRISE TO CARPET HIS DEN?

ADVERTISING PUZZLES

(DISREGARD SALES TAX)

1) ELIZABETH'S MOTHER SAID THAT SHE CAN SELECT SOME NEW CLOTHES FOR THE REST OF THE SCHOOL YEAR. SHE DECIDED ON 2 LONG SLEEVE TOPS, 2 SWEATERS, AND 2 SKIRTS. HOW MUCH DID THEY COST?

2) SUSAN'S MOTHER TOLD HER THE SAME THING. SHE DECIDED ON 3 PAIRS OF PANTS, 1 SWEATER, 2 BLOUSES, AND 1 SKIRT. HOW MUCH DID THEY COST?

3) SUSAN'S CLOTHES COST HOW MUCH MORE THAN ELIZABETH'S CLOTHES?

4) DO YOU THINK ELIZABETH WAS MAD BECAUSE SUSAN'S CLOTHES COST MORE THAN HERS? EXPLAIN YOUR ANSWER.

5) IF LOUISE HAS $10.00, WHAT IS THE LARGEST NUMBER OF ARTICLES THAT SHE CAN BUY FROM THE LIST ABOVE?

ADVERTISING PUZZLES

(DISREGARD SALES TAX)

1) DICK AND SALLY ARE GOING SKIING WITH THEIR PARENTS FOR THE FIRST TIME. THEY EACH NEED GLOVES, GOGGLES, JACKETS, AND BOOTS. SALLY WANTS THE MOST EXPENSIVE JACKET AND WILL SETTLE FOR THE CHEAPEST GLOVES. WHAT WILL HER THINGS COST?

2) DICK WANTS THE MOST EXPENSIVE GLOVES AND THE LEAST EXPENSIVE JACKET. WHAT WILL HIS THINGS COST?

3) SALLY'S CLOTHES COST HOW MUCH MORE THAN DICK'S CLOTHES?

4) DICK WANTS A PAIR OF GLOVE LINERS AND 3 PAIR OF SKI SOX IF THERE IS ENOUGH DIFFERENCE IN PROBLEM THREE. IS THERE?

76

FINDING A SQUARE ROOT OF A NUMBER

TO SQUARE A NUMBER IS TO USE IT AS A FACTOR TWICE. FOR EXAMPLE, THE SQUARE OF $3 \times 3 = 9$.

FINDING A SQUARE ROOT OF A NUMBER IS TO FIND ONE OF ITS TWO EQUAL FACTORS. FOR EXAMPLE, A SQUARE ROOT OF 9, WRITTEN $\sqrt{9}$, IS 3 BECAUSE $3 \times 3 = 9$. FINDING A SQUARE ROOT OF A NUMBER IS THE INVERSE (OPPOSITE) OPERATION OF SQUARING.

TO INDICATE A SQUARE ROOT OF A NUMBER, A RADICAL SIGN, $\sqrt{}$, IS USED. THE SYMBOL $\sqrt{9}$ IS CALLED A RADICAL; 9, THE NUMBER UNDER THE RADICAL SIGN, IS CALLED THE RADICAND.

NUMBERS	PERFECT SQUARES	SQUARE ROOTS
1	1	1
2	4	2
3	9	3
4	16	4
5	25	5
6	36	6
7	49	7
8	64	8
9	81	9

NUMBER SENSE

MULTIPLY BY 5

TAKE HALF OF THE NUMBER AND MULTIPLY BY 10.

36	THE NUMBER	36
18	HALF OF THE NUMBER	x5
180	MULTIPLY BY 10	180

47	THE NUMBER	47
23.5	HALF OF THE NUMBER	x5
235	MULTIPLY BY 10	235

MULTIPLY BY 15

MULTIPLY THE NUMBER BY 10 AND ADD HALF THE PRODUCT TO IT.

49	THE NUMBER	49
490	MULTIPLY BY 10	15
245	HALF THE PRODUCT	245
735	SUM OF PRODUCT AND	49
	HALF THE PRODUCT	735

MULTIPLY BY 9

MULTIPLY THE NUMBER BY 10 AND SUBTRACT THE NUMBER FROM IT.

93	THE NUMBER	93
930	MULTIPLY BY 10	x9
837	SUBTRACT THE NUMBER	837

MULTIPLY BY 99

MULTIPLY THE NUMBER BY 100 AND SUBTRACT FROM THIS PRODUCT THE ORIGINAL NUMBER.

236	THE NUMBER	236
23600	MULTIPLY BY 100	x99
23364	SUBTRACT THE NUMBER	2124
		2124
		23364

MULTIPLY BY 11

1. WRITE THE RIGHT-HAND DIGIT OF THE NUMBER AS THE RIGHT-HAND DIGIT OF THE ANSWER.
2. ADD TWO DIGITS AT A TIME STARTING AT THE RIGHT TO PRODUCE THE SUCCEEDING DIGITS OF THE ANSWER.
3. WRITE THE LEFT-HAND DIGIT OF THE NUMBER AS THE LEFT-HAND DIGIT OF THE ANSWER.

32415 x 11	32415
356565	x11
	32415
	32415
	356565

START ON THE RIGHT AND WORK LEFT.

WRITE THE RIGHT-HAND DIGIT (5).

ADD 5 + 1 = 6 FOR THE NEXT DIGIT.

ADD 1 + 4 = 5 FOR THE NEXT DIGIT.

ADD 4 + 2 = 6 FOR THE NEXT DIGIT.

ADD 2 + 3 = 5 FOR THE NEXT DIGIT.

WRITE THE LEFT-HAND DIGIT (3).

4. Sometimes it is necessary to carry.

   257861 x 11 = 2836471

   Start on the right and work left.

   Write the right-hand digit (1).

   Add 1 + 6 = 7 for the next digit.

   Add 6 + 8 =14; write 4 and carry 1.

   Add 8 + 7 =15; 15 + 1 = 16; write 6 and carry 1.

   add 7 + 5 =12; 12 + 1 = 13; write 3 and carry 1.

   add 5 + 2 = 7; 7 + 1 = 8; write 8

   Write 2 for the last digit on the left.

Multiply by 12

1. Double the right-hand digit of the number to be the right-hand digit of the answer.
2. Double each succeeding digit and add to the digit on the right plus anything that is carried.
3. The left-hand digit of the number plus anything that is carried becomes the left-hand digit of the answer.

   32761 x 12 = 393132

   Double 1, write 2.

   Double 6, add 1, write 3, carry 1.

   Double 7, add 6, add 1, write 1, carry 2.

   Double 2, add 7, add 2, write 3, carry 1.

   Double 3, add 2, add 1, write 9.

   Write 3.

MULTIPLY TWO-DIGIT NUMBER BY TWO-DIGIT-NUMBER

$^{6}39$
x47

1. START MULTIPLYING AS USUAL WITH THE ONES PLACES.

3

2. STOP USING THE REGULAR METHOD.

$^{6}3\ 9$
x 4 7

3. NEXT, CROSS MULTIPLY.

3 x 7 = 21
9 x 4 = 36

4. ADD THE PRODUCTS.

57
6

5. ADD THE ONES CARRIED.

63

6. WRITE THE 3 IN THE TENS PLACE IN THE ANSWER, AND CARRY 6.

$^{6\ 6}39$
x47
33
3 x 4 = 12

7. MULTIPLY THE TENS PLACES, ADD THE ONES CARRIED, AND WRITE THIS TO COMPLETE THE ANSWER.

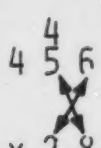
$^{6\ 6}39$
x47
1833

MULTIPLY A THREE-DIGIT NUMBER BY A TWO-DIGIT NUMBER

$$\begin{array}{r} {\scriptstyle 4} \\ 456 \\ \times 28 \\ \hline 8 \end{array}$$

1. START MULTIPLYING AS USUAL WITH THE ONES PLACE.

2. STOP USING THE REGULAR METHOD.



3. NEXT, CROSS MULTIPLY AS SHOWN.

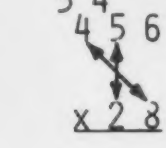
$$\begin{array}{r} 8 \\ 5 \times 8 = 40 \\ 2 \times 6 + 12 \\ \hline 52 \\ 4 \\ \hline 56 \end{array}$$

4. ADD THE PRODUCTS.

5. ADD THE ONES CARRIED.

$$\begin{array}{r} {\scriptstyle 54} \\ 456 \\ \times 28 \\ \hline 68 \end{array}$$

6. WRITE THE 6 IN THE TENS PLACE IN THE ANSWER, AND CARRY 5.



7. NEXT, CROSS MULTIPLY AS SHOWN.

$$\begin{array}{r} 6\ 8 \\ 2 \times 5 = 10 \\ 4 \times 8 = 32 \\ \hline 42 \\ 5 \\ \hline 47 \end{array}$$

8. ADD THE PRODUCTS.

9. ADD THE ONES CARRIED.

454
456
28
768

454
4 5 6
2 8
7 6 8
4 x 2 = 8
4
12

454
456
28
12768

10. WRITE THE 7 IN THE HUNDREDS PLACE AND CARRY 4.

11. MULTIPLY THE HUNDREDS PLACE BY THE TENS PLACE.

12. ADD THE ONES CARRIED.

13. WRITE THIS TO COMPLETE THE ANSWER.

MULTIPLY NUMBERS BY 25

DIVIDE THE NUMBER BY 4 AND MULTIPLY THE QUOTIENT BY 100.

25×48 DIVIDE 48 BY 4 ⟶ 12.

$\frac{100}{4} \times 48$ MULTIPLY 12 BY 100 ⟶ 1200.

100×12

1200

MULTIPLY NUMBERS BY 50

DIVIDE THE NUMBER BY 2 AND MULTIPLY THE QUOTIENT BY 100.

50×74 DIVIDE 74 BY 2 ⟶ 37.

$\frac{100}{2} \times 74$ MULTIPLY 37 BY 100 ⟶ 3700.

100×37

3700

MULTIPLY NUMBERS BY 75

DIVIDE THE NUMBER BY 4, MULTIPLY THE QUOTIENT BY 3, AND MULTIPLY THE PRODUCT BY 100.

75×28

$100 \times \frac{3}{4} \times 28$ DIVIDE 28 BY 4 ⟶ 7.

$100 \times 3 \times 7$ MULTIPLY 7 BY 3 ⟶ 21.

100×21 MULTIPLY 21 BY 100 ⟶ 2100.

2100

MULTIPLY NUMBERS CLOSE TO 100

93 x 97

1. TAKE THE COMPLEMENTS TO 100 OF BOTH NUMBERS AND MULTIPLY THEM.

   93--7 97--3

   7 x 3 =21

__ __ 2 1

2. WRITE 21 IN THE TEN'S AND ONE'S PLACES OF THE ANSWER.

93-3 = 90

OR

97-7 = 90

3. SUBTRACT THE COMPLEMENT OF ONE NUMBER FROM THE OTHER NUMBER AND PLACE THE RESULT NEXT TO 2 1.

9 0 2 1

SQUARING A NUMBER

$(47)^2$

1. SQUARE 7 TO GET 49.

9

2. PUT THE 9 IN THE ONE'S COLUMN OF THE ANSWER AND CARRY THE 4.
3. TAKE 4 x 7 x 2 = 56 AND ADD THE 4 FROM STEP 2 TO GET 60.

0 9

4. PUT THE 0 IN THE TEN'S COLUMN OF THE ANSWER AND CARRY THE 6.
5. SQUARE 4 TO GET 16 AND ADD THE 6 FROM STEP 4 TO GET 22.

2 2 0 9

6. PLACE THIS RESULT NEXT TO 0 9.

Short cut for squaring numbers 51-59

$(53)^2$

1. Write 3^2 as a two-digit number. 0 9
2. Square 5. 25
3. Add 3. 3
   28
4. Answer 2 8 0 9

Short cut for squaring a number ending in 5

$(75)^2$

1. Write 5^2 in one's and ten's place. 2 5
2. Multiply 7 by its successor 8. 5 6
3. Answer 5 6 2 5

Multiply by Difference of 2 Squares Method

47 x 53 is
(50-3) x (50+3)
which is

$50^2 - 3^2$

1) Square 50. 2500
2) Square 3. 9
3) Subtract. 2491

Numbers must "center" around a number like 40, 50, 100, etc.

$74 \times 86 = (80 - 6)(80 + 6) = 80^2 - 6^2 = 6400 - 36 = 6364$

Using Difference of 2 Squares for other problems

$$64^2 - 36^2 = (64+36)\ (64-36) = (100)\ (28) = 2800$$

Divisibility by 2

A number is divisible by 2 if the last digit is even (0, 2, 4, 6, or 8).

Divisibility by 3

A number is divisible by 3 if the sum of its digits is divisible by 3.

267 Sum of its digits: 2 + 6 + 7 = 15.

Since 15 is divisible by 3, then 267 is divisible by 3.

Divisibility by 4

A number is divisible by 4 if its last two digits form a number that is divisible by 4.

624 Since 24 is divisible by 4, then 624 is divisible by 4.

737 Since 37 is <u>not</u> divisible by 4, then 737 is not divisible by 4.

Divisibility by 5

A number is divisible by 5 if the last number is 0 or a 5.

Divisibility by 6

A number is divisible by 6 if it is divisible by 2 and by 3.

366 is divisible by 6 because it is divisible by 2 (the last digit 6 is even) and it is divisible by 3 (the sum of the digits is 15).

Divisibility by 7

Double the last digit of the numbers and subtract it from the remaining digits. If the remaining number is divisible by 7, the original number is divisible by 7. If the second number is still too large to divide by inspection, continue doubling the last digit and subtracting from the remaining digits until the answer is a number that is small enough to divide by inspection.

238 Double 8 and subtract the result from the remaining digits. Since 7 is divisible by 7, then 238 is divisible by 7.

-16

7

4327 Double 7 and subtract the results from the remaining digits.

-14

418

16 Double 8 and subtract the results from the remaining digits.

25

Since 25 is not divisible by 7, then 4327 is not divisible by 7.

Divisibility by 8

A number is divisible by 8 if the last three digits are divisible by 8.

4234 Since 234 is divisible by 8, then 4234 is divisible by 8.

62311 Since 311 is not divisible by 8, then 62311 is not divisible by 8.

88

Divisibility by 9

A number is divisible by 9 if the sum of its digits is divisible by 9.

261 — Sum of the digits: 2 + 6 + 1 = 9 which is divisible by 9. Therefore, 261 is divisible by 9.

Divisibility by 10

A number is divisible by 10 if the last digit is 0.

Divisibility by 11

1. Starting with the one's place, place an "x" over every other digit in the number.
2. Add the digits with an "x" above them.
3. Add the digits without an "x" above them.
4. Find the difference of the two sums.
5. If the difference is divisible by 11, then the number is divisible by 11.

$\overset{x}{4}\ 3,\ \overset{x}{5}\ 5\ \overset{x}{6}$ — $4 + 5 + 6 = 15$; $3 + 5 = 8$; $15 - 8 = 7$. This number is not divisible by 11.

$\overset{x}{9}\ 8,\ \overset{x}{6}\ 1\ \overset{x}{5}$ — $9 + 6 + 5 = 20$; $8 + 1 = 9$; $20 - 9 = 11$. This number is divisible by 11.

Divisibility by 25

A number is divisible by 25 if the last two digits are 00, 25, 50, or 75.

Divisibility by 50

A number is divisible by 50 if the last two digits are 00 or 50.

NUMBER SENSE

MULTIPLICATION

1)	11	X	25	=	16)	12	X	25	=
2)	11	X	17	=	17)	12	X	17	=
3)	11	X	108	=	18)	12	X	108	=
4)	11	X	67	=	19)	12	X	67	=
5)	11	X	125	=	20)	12	X	125	=
6)	11	X	1362	=	21)	12	X	1362	=
7)	11	X	88	=	22)	12	X	88	=
8)	11	X	169	=	23)	12	X	169	=
9)	11	X	1095	=	24)	12	X	1095	=
10)	11	X	72	=	25)	12	X	72	=
11)	11	X	66	=	26)	12	X	66	=
12)	11	X	862	=	27)	12	X	862	=
13)	11	X	1776	=	28)	12	X	1776	=
14)	11	X	556	=	29)	12	X	556	=
15)	11	X	1932	=	30)	12	X	1932	=

NUMBER SENSE

MULTIPLICATION

1) 25 X 16 =

2) 25 X 836 =

3) 25 X 72 =

4) 25 X 40 =

5) 25 X 20 =

6) 25 X 12 =

7) 25 X 280 =

8) 50 X 24 =

9) 50 X 76 =

10) 50 X 36 =

11) 50 X 88 =

12) 25 X 88 =

13) 25 X 24 =

14) 25 X 32 =

15) 25 X 48 =

16) 25 X 44 =

17) 50 X 44 =

18) 25 X 104 =

19) 50 X 52 =

20) 75 X 12 =

21) 75 X 32 =

22) 75 X 64 =

23) 75 X 48 =

24) 75 X 20 =

25) 75 X 24 =

26) 75 X 28 =

27) 11 X 431 =

28) 11 X 26 =

29) 11 X 817 =

30) 11 X 1234 =

DIVISIBILITY

PLACE AN "X" IN THE BOX IF THE NUMBER IN THE LEFT COLUMN IS DIVISIBLE BY THE NUMBER IN THE TOP ROW.

	2	3	4	5	6	7	8	9	10	11	25	50
572												
75867												
142142												
3632												
4566												
24												
729												
644												
201												
1560												
63												
162												
255												
11101												
78475												
23500												
625												
19836												
3336												
6162												
138600												

ANSWERS

MATHEMATICS ENRICHMENT
GRADE FOUR
ANSWERS

Page 6

1. 20.75 bu.
2. a) 643.5 lb.
   b) 64,350 lb.
3. $.98
4. a) $.52
   b) $519.00
5. $25.32
6. a) 3.67 lb.
   b) 367 lb.
7. a) 13,400 lb.
   b) 134,000 lb.
   c) 1,340,000 lb.
8. $1.80

Page 7

1. a) $.40
   b) $.87
2. $70
3. $2.16
4. $490
5. $12.25
6. a) $20
   b) $.50
7. $1575

Page 8

1. $8.17
2. 1888
3. 1788
4. 64 8
5. a) $12
   b) $30
6. a) $675
   b) $450
   c) $1850

Page 9

1. 8
2. 9
3. 7
4. 2
5. 10
6. 3
7. 4
8. 5
9. 6

Page 10

1. $3
2. $10
3. 31
4. 9
5. 8
6. 7
7. 29
8. 30
9. 32 ft.

Page 11

1. $5.04
2. 581
3. $180
4. 303
5. 6
6. 500
7. a) $3.56
   b) $.31
   c) $3.94

Page 12

1. a) $1.78
   b) $16.11
2. a) 7084
   b) 990 km
3. $1.50
4. $2.20
5. 4
6. a) $292
   b) $272

Page 13

1. a) 19 cm
   b) 7 cm
2. a) 455 km
   b) 4543 km
3. a) $1.58
   b) $1.52
4. $13.50
5. a) 101
   b) 1433

Page 14

1. a) 60 km
   b) 2 km
2. a) 1789 km
   b) 768 km
3. a) 4_2
   b) 4^2
4. a) $4.28
   b) $6.27
5. $.96

Page 15

No map has been discovered yet which cannot be colored with four or fewer colors.

Page 30

1968

Page 33

1. Luke

Page 34

2. Numbers 16 and 31
3. A half dollar, quarter, and four dimes
4. $8.75
5. Three socks. If the first two fail to match, then the third is sure to match one of the two.

Page 35

6. If Dad keeps his agreement, he will have to pay $10,737,418.24.

7. The farmer must take the goose and leave it on the far side. Then he must take the corn and return with the goose, leaving the corn on the other side. Now he leaves the goose and takes the fox across and leaves it with the corn. Next, he returns with the goose.

Page 36

8. He made money. His profit was $20.00.

9. She fills the five gallon tank from the eight gallon tank. She then fills the three gallon tank from the five. She pours the three into the eight, puts the two from the five into the three gallon tank, and then fills the five gallon tank from the eight gallon tank. Then she fills the three gallon tank from the five gallon tank, leaving four gallons in it. Next the three gallon tank is poured into the eight gallon tank, and with the one gallon already left in it, it now contains four gallons.

10. 28 days. On the 28th day he would climb and not slip back.

11. Mrs. B and Mrs. C cross, and Mrs. B returns with the boat. Mrs. A and Mrs. B cross and Mrs. A returns with the boat. Mr. B and Mr. C. cross, and Mr. and Mrs. B return. Mr. A and Mr. B cross, and Mrs. C returns with the boat. Mrs. B and Mrs. C cross, and Mr. A returns to pick up his wife.

12. 8 days, since on that day he will reach the top and will not slide back down.

13. Ramon is 10. Juanita is 12. Marie is 13. Juan is 14.

14. Ms. Arteaga bought 32 oz.; Ms. Boyd bought 8 oz.; Ms. Cameron bought 16 oz.

15. Al's dog is the dachshund and it took second prize. Ben's dog is the boxer and it took first prize. Charles' dog is the collie and it took third prize.

16. Barbara prefers hiking; Howell prefers tennis; Irene prefers baseball; Tona perfers ice-skating.

Page 38

17. George lives in Ohio and drinks lemonade. Lewis lives in Georgia and drinks orange juice. Oscar lives in Louisiana and drinks grape juice.

18. First, white; then pink, green, and yellow.

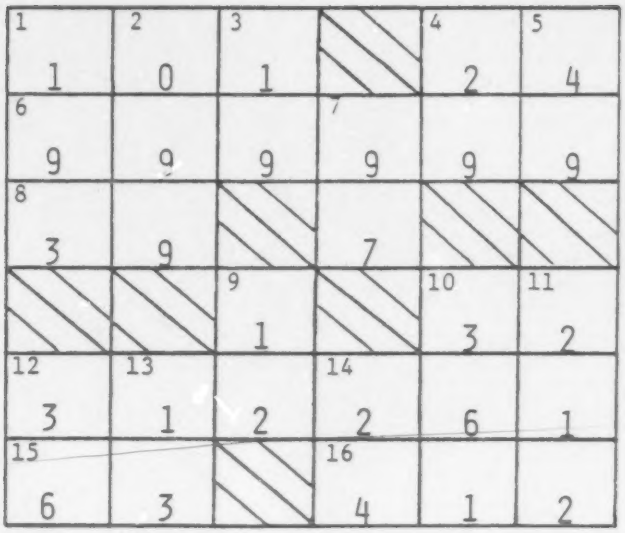
19. Dewey: 22°C; William: 23; Arthur: 24.

Page 39

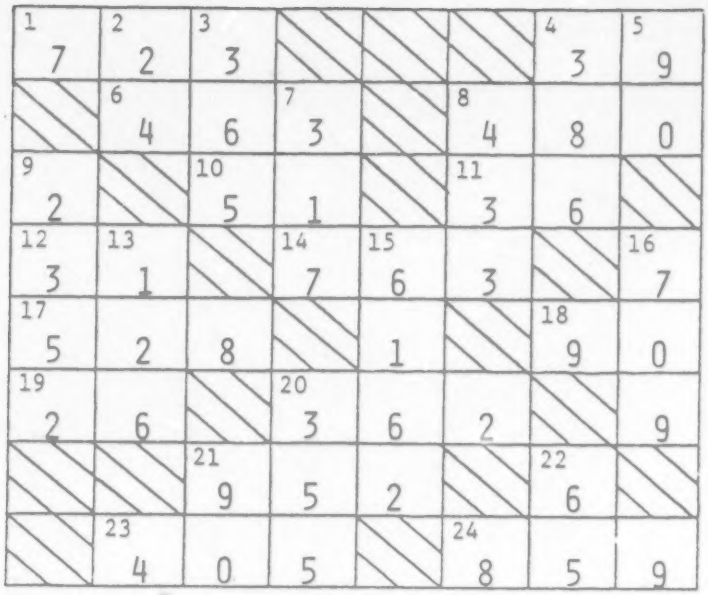
20. Frank had four coins; Ramiro had five coins; Harold had two.

21. $1 Washington, $5 Lincoln, $10 Hamilton, $20 Jackson, $50 Grant, $100 Franklin, $500 McKinley, $1,000 Cleveland, $5,000 Madison, and $10,000 Chase.

Page 40



Page 41



Page 42

1. Quarter

Page 43

1.
$$\begin{array}{r} 9 \\ 8 \\ 983 \\ \hline 1000 \end{array}$$

2.
$$\begin{array}{r} 2008 \\ 4031 \\ 3960 \\ \hline 9999 \end{array}$$

3.
$$\begin{array}{r} 122 \\ 67 \\ 443 \\ \hline 632 \end{array}$$

4.
$$\begin{array}{r} 10008 \\ 9999 \\ \hline 9 \end{array}$$

5.
$$\begin{array}{r} 9335 \\ 7263 \\ \hline 2072 \end{array}$$

6.
$$\begin{array}{r} 97324 \\ 79518 \\ \hline 17806 \end{array}$$

7.
$$\begin{array}{r} 14582 \\ 505 \\ \hline 72910 \\ 72910 \\ \hline 7363910 \end{array}$$

8.
$$\begin{array}{r} 239 \\ 703 \\ \hline 717 \\ 16730 \\ \hline 168017 \end{array}$$

9.
$$\begin{array}{r} 53 \\ 96 \\ \hline 318 \\ 477 \\ \hline 5088 \end{array}$$

10.
$$\begin{array}{r} 303 \\ 579\overline{)175437} \\ 1737 \\ \hline 1737 \\ 1737 \\ \hline \end{array}$$

11.
$$\begin{array}{r} 17 \\ 563\overline{)9571} \\ 563 \\ \hline 3941 \\ 3941 \\ \hline \end{array}$$

12.
$$\begin{array}{r} 10409 \\ 85\overline{)884765} \\ 85 \\ \hline 347 \\ 340 \\ \hline 765 \\ 765 \\ \hline \end{array}$$

Page 44

1. 2,234,822
2. 4,984,097
3. common fractions
4. 2 hours
5. 5 hours

Page 45

6. $\frac{1}{4}$
7. $\frac{1}{8}$
8. 9 months
9. $7\frac{3}{8}$
10. 512 feet
11. 32
12. $99
13. $81
14. $8

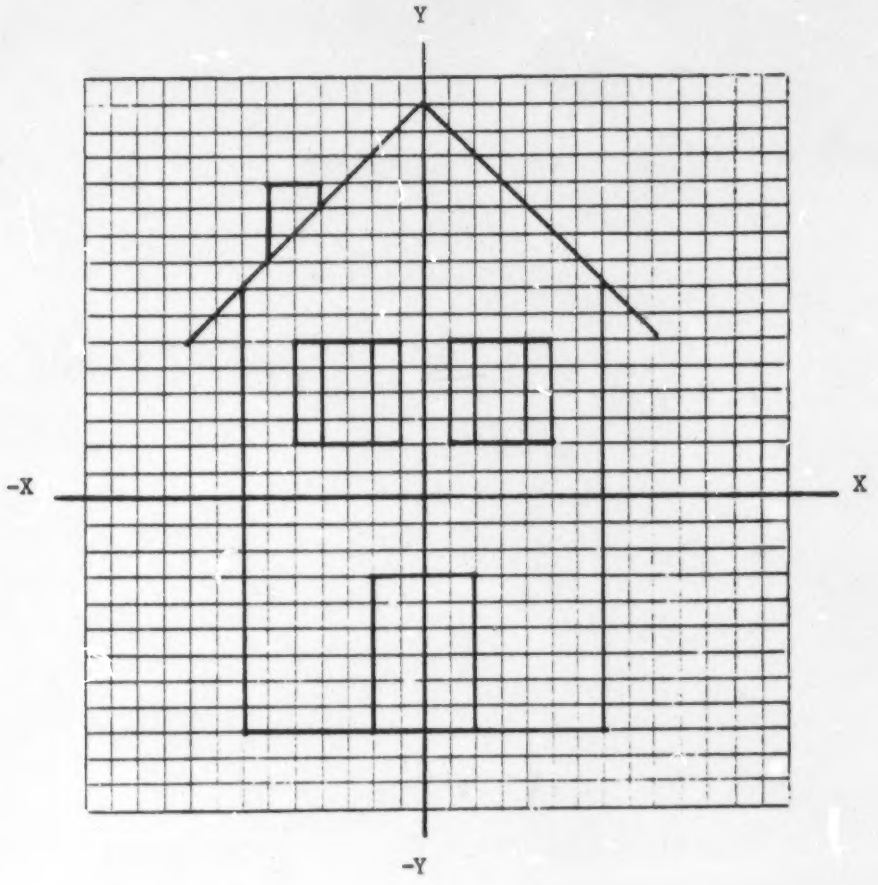
Page 46

15. $700
16. $2.40
17. $\$ 3\frac{1}{8}$
18. 3 hr. 51 min. 11 sec.
19. 2 miles 860 feet
20. $250

97

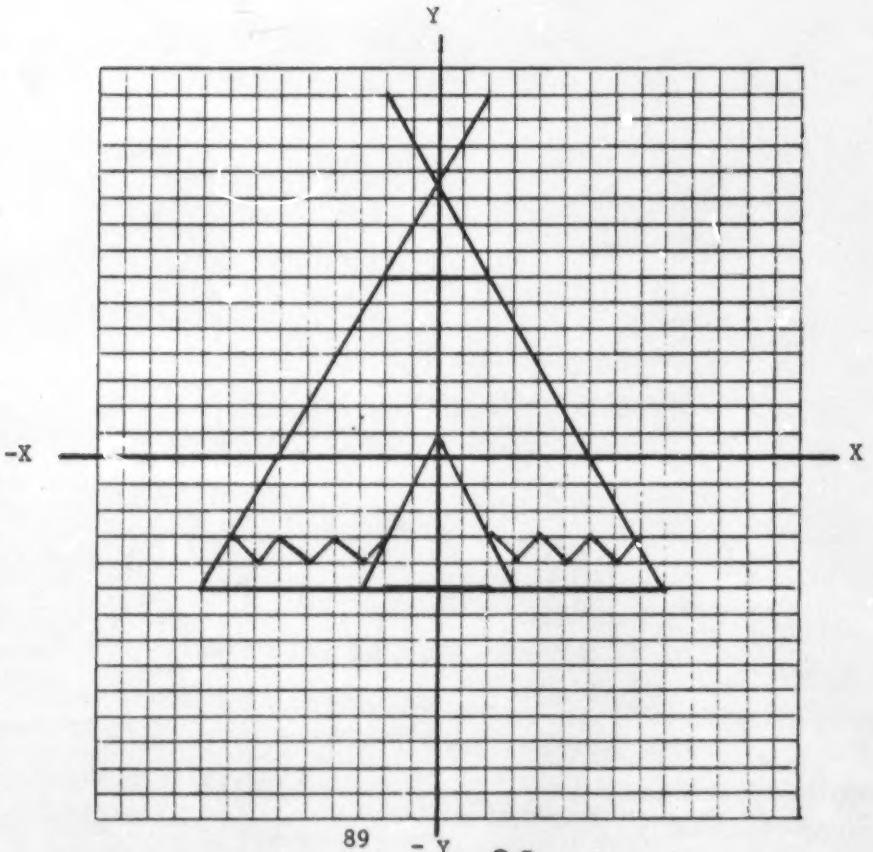
Page 50

1.



House

2.

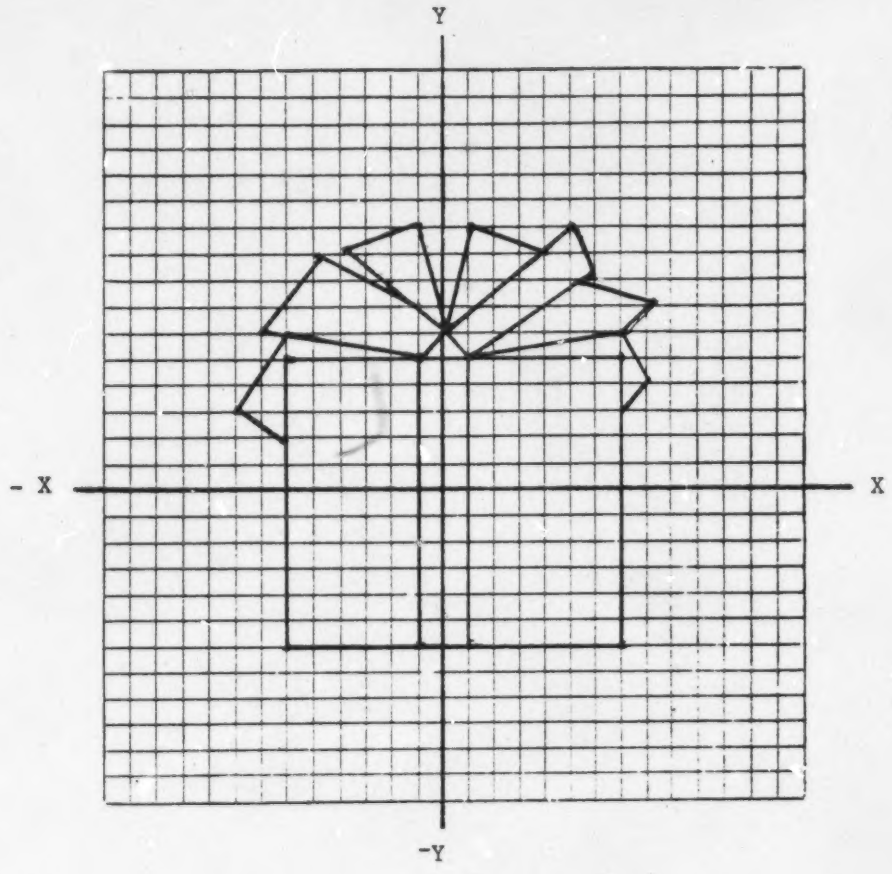


Tepee

Page 50

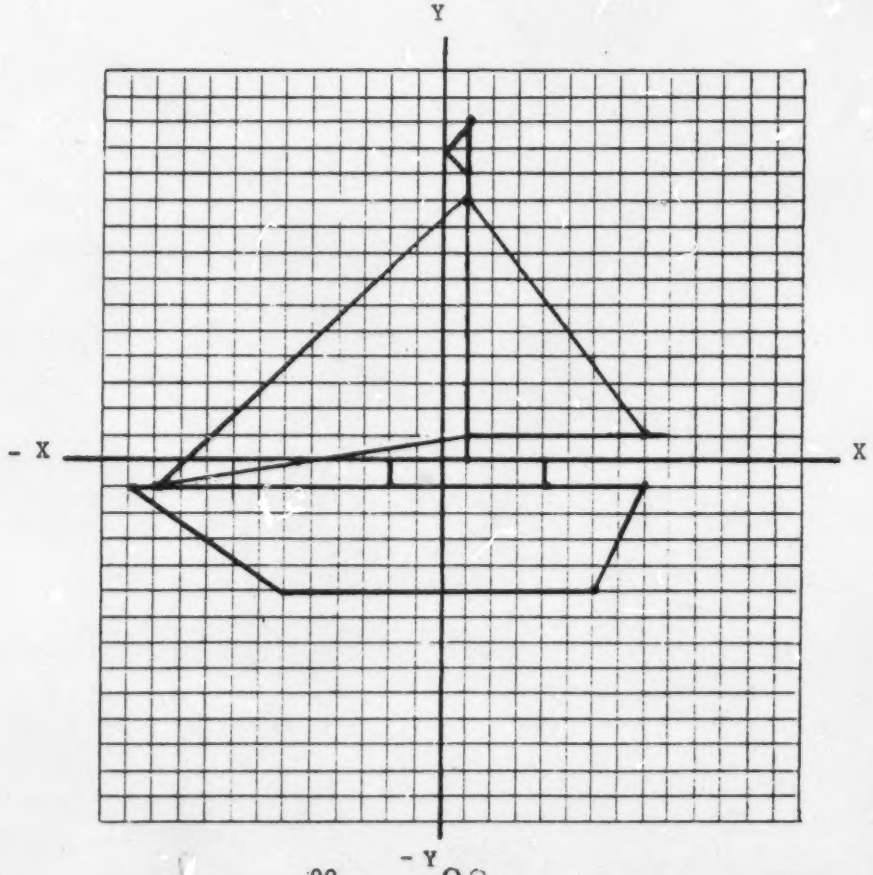
3.

Package



Page 51

4.



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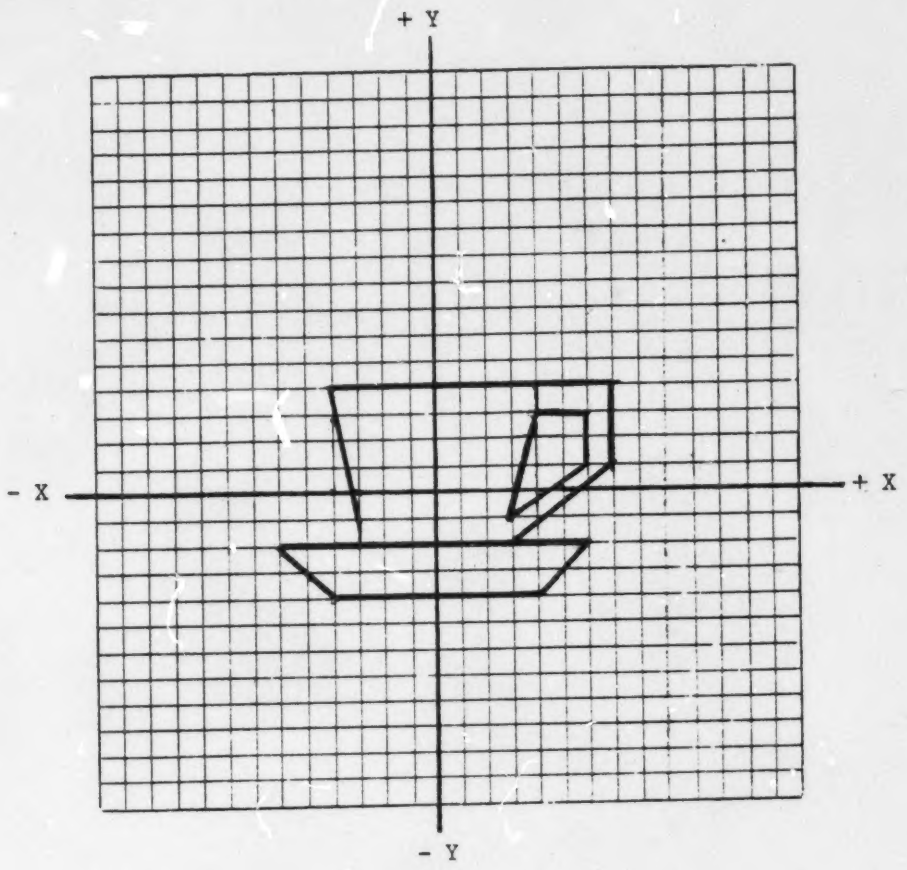
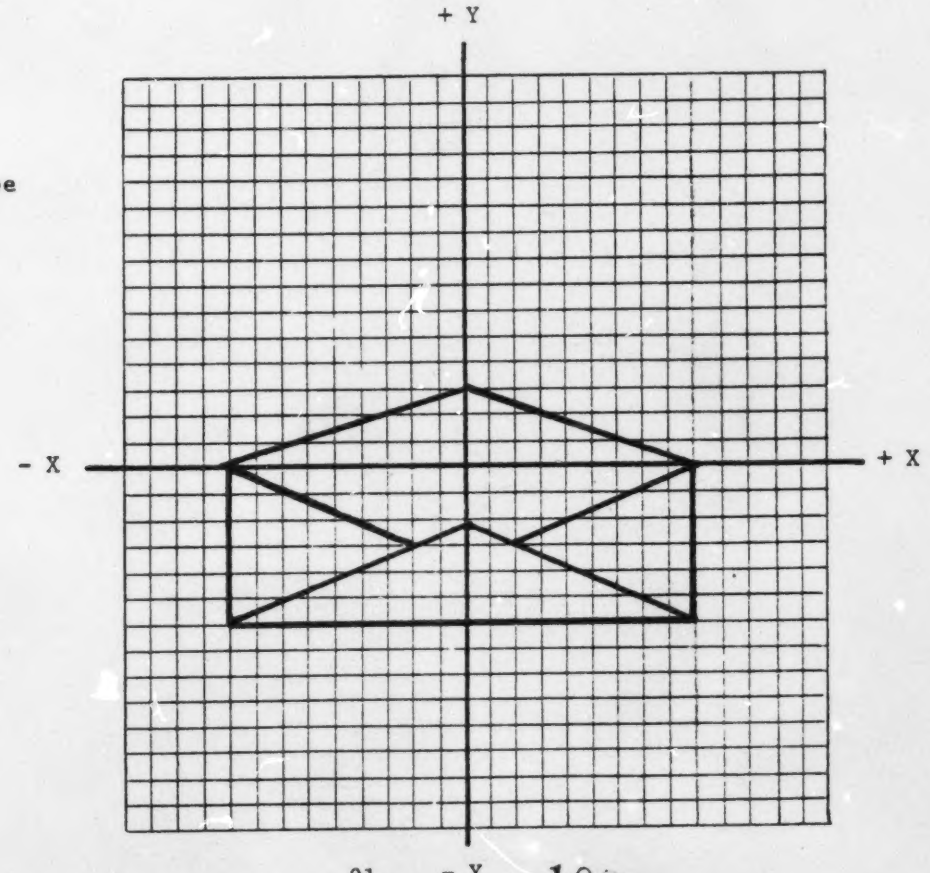
5.

Cup and Saucer



6.

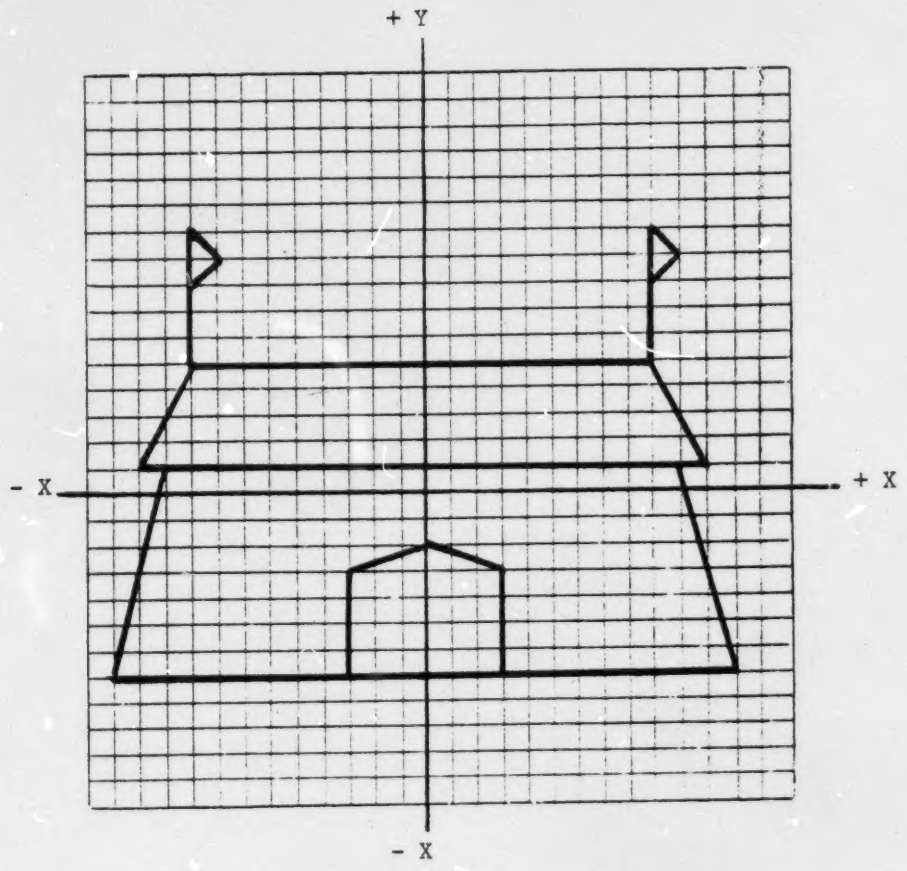
Envelope



Page 51

7.

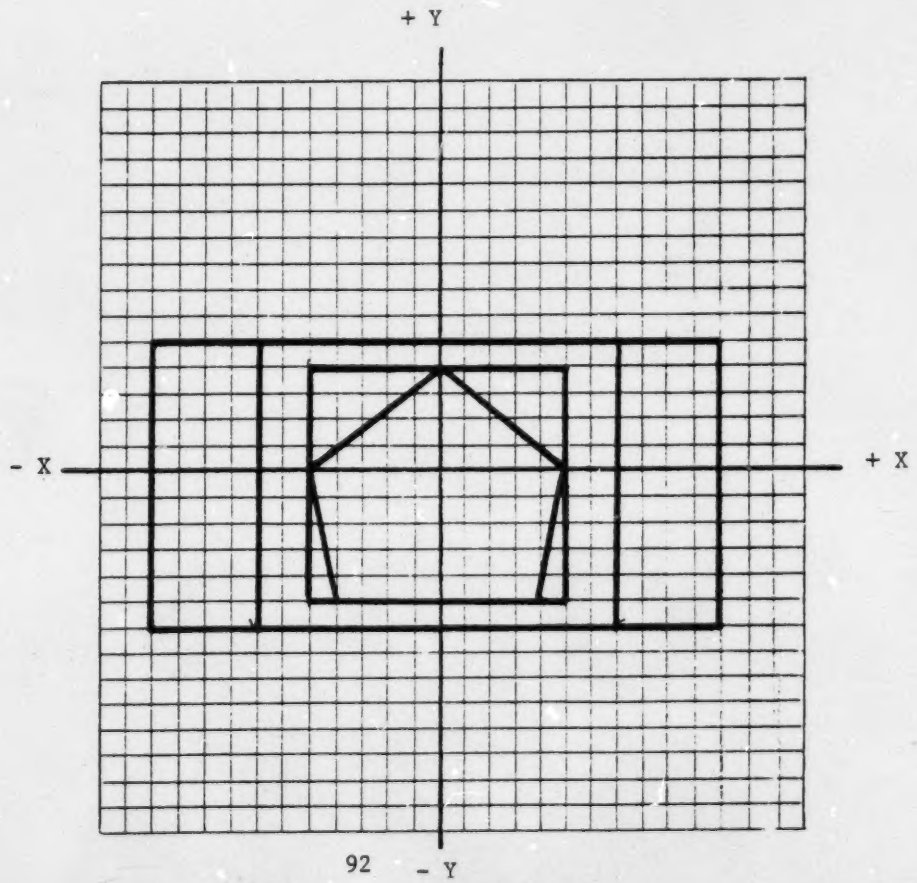
Circus
Tent



Page 52

8.

Window



Page 54

1. 8 triangles
   3 rectangles
2. 12 triangles
   9 rectangles
3. 24 triangles
   6 squares
   33 rectangles

Page 55

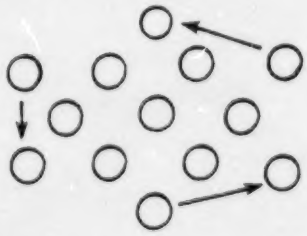
3 squares
6 rectangles
10 triangles
17 quadrilaterals

Page 56

16 squares
39 rectangles
36 triangles
4 cubes

Page 57

28 rectangles
96 triangles (at least)
121 quadrilaterals (at least)



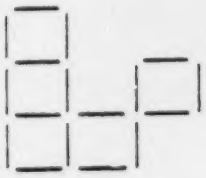
Page 58

1.

Page 58 (cont'd)

2.

3.



4.

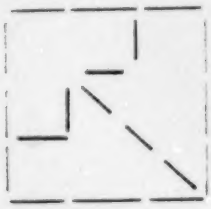
Page 59

1.

2.

Page 59 (cont'd)

3.

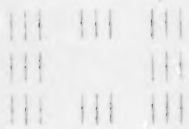


4.

Page 60

1.

2.



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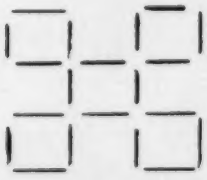
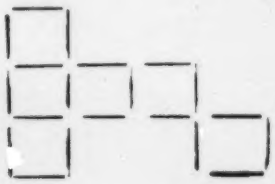


Page 60 (cont'd)

4.



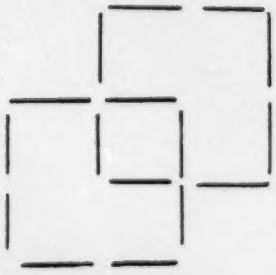
Page 61

1.

2.

3.

4.



Page 65

1. $154.24
2. Insufficient information. She must decide how many franks to allow for each girl.
3. 70
4. $7.14
5. The family pack will be cheaper.

Page 66

1. $389.85
2. $251.85
3. $138.00
4. $239.20
5. colorfair

Page 67

1. $39.[illegible]
2. $39.96
3. 2c
4. No--almost the same cost
5. 5

Page 68

1. $155.80
2. $95.80
3. $60.00
4. yes

Page 82

1. 275
2. 187
3. 1188
4. 737
5. 1375
6. 14982
7. 968
8. 1859
9. 12045
10. 792
11. 726
12. 9482
13. 19536
14. 6116
15. 21252
16. 300
17. 204
18. 1296
19. 804
20. 1500
21. 16344
22. 1056
23. 2028
24. 13140
25. 864

Page 82 (cont'd)

26. 792
27. 10344
28. 21312
29. 6672
30. 23184

Page 83

1. 400
2. 20900
3. 1800
4. 100
5. 500
6. 300
7. 7000
8. 1200
9. 3800
10. 1800
11. 4400
12. 2200
13. 600
14. 800
15. 1200
16. 1100
17. 2200
18. 2600
19. 2600
20. 900
21. 2400
22. 5800
23. 3600
24. 1500
25. 1800
26. 2100
27. 4741
28. 286
29. 8987
30. 13574

DIVISIBILITY

	2	3	4	5	6	7	8	9	10	11	25	50
572	X	X	X		X					X		
75867		X								X		
142142	X					X				X		
3632	X		X				X					
4566	X	X			X							
24	X	X	X		X		X					
729		X						X				
644	X		X			X						
201		X										
1560	X	X	X	X	X		X		X			
63		X				X		X				
162	X	X			X			X				
255		X		X								
11101												
78475				X							X	
23500				X					X		X	X
625				X							X	
19836	X	X	X		X			X				
3336	X	X	X		X		X					
6162	X	X			X							
138600	X	X	X	X	X	X	X	X	X	X	X	X